

# Constraint Relaxation on Topological Operators Which Produce a Null Value as Answer to a Query

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## ABSTRACT

*In the field of spatial information systems, and especially in particular of geographical databases, many authors have studied how to formulate queries using pictorial configurations. The constraints deriving from topological relationships between pairs of symbolic graphical objects (the classic shapes point, polyline, and polygon) can be relaxed when a query search condition finds no match in the database, so that users can receive approximate answers rather than null information. In this paper a computational model for the similarity of the spatial relations is proposed. An operator conceptual similarity (OCS) graph describes the model, and links the more similar relations, defining the weight of each relaxation. The polygon-polygon, polyline-polyline, and polygon-polyline cases are discussed and matrices, which enlarge the known 9-Intersection model matrix are also considered.*

## 1. INTRODUCTION

Many researchers have recently focused their attention on different approaches to expressing geographical data queries. The evolution of visual query languages has led to the proposal of pictorial query representation. Furthermore, computer networks and distributed computing technology have transformed many computer-based applications from the traditional stand-alone mode to the networked mode.

Geographical databases have received considerable attention due to the emergence of new applications. These databases too, some research efforts have focused on the problem of human-computer interaction and the representation of visual queries for geographical data.

In the field of spatial databases, many authors have studied how to formulate queries using pictorial configurations. In a geographical database these enable the configuration of geographical objects to be described, thus expressing the user's "mental model" of the query [4, 13].

The user's mental model corresponds to the semantics of the query (in the user's mind). It may show some mismatching with the Visual Sentence for the query. Let us suppose the user wishes to express the query that has the following textual description:

*"Find all the regions passed through by a river and partially overlapping a forest"*

The user is not interested in the relationship between the river and the forest; however the absence, in natural language formulation, of any explicit relation-

ship between them produces ambiguity. In fact, the user implicitly thinks that no relationship exists between any river and any forest, but the correct query expression without any ambiguity using natural language is:

*"Find all the regions passed through by a river and partially overlapping a forest, irrespective of the topological relationships between the river and the forest".*

To represent this query graphically some authors use the classic shapes "point, polyline, and polygon". In [4, 12] the authors propose a pictorial query language called GeoPQL (Geographical Pictorial Query Language) to express any queries, and call these shapes Symbolic Graphical Objects (SGO) [4, 12]. The topological relationships between pairs of SGO use operators based on the Object-Calculus proposed in [14, 15], where a set of topological, metric and logical operators are defined. When a query search condition finds no match in the database, it would be useful for the system to produce not only configurations with an exact correspondence with the drawing representing the pictorial query, but also similar configurations obtained by relaxing some of the constraints. The most common approach to relax constraints is to measure the distance from the drawn query using criteria defined for the specific domain. In the case of pictorial queries of a geographical database, the constraints can be classified as three main types: spatial, structural and semantic constraints.

In this paper we discuss spatial constraints [5, 6], in order to decide which constraints must be relaxed and which maintained. To do this, we define a computational model for the similarity of the spatial relations by which to transform the pictorial query.

The similarity between topological relations is described by the **Operator Conceptual Similarity graph** (OCS graph), which links the most similar operators (in the sense explained in the following section), defining the weight of each relaxation. The query answers are produced by assigning a total score computed by this computational model.

In recent years, various papers have discussed the problems with topological relations between pairs of objects in a 2-dimensional space. Two models for binary topological relations - the 4-Intersection model and the 9-Intersection model - have been proposed [1, 2, 3] and compared [7]. In the 9-intersection model, as described in [8], the interior ( $A^\circ$ ), boundary ( $\partial A$ ), and exterior ( $A^-$ ) of a 2-dimensional point set embedded in  $\mathbb{R}^2$  are defined as usual and will be referred to as the topological parts of an object. The definition of binary topological relationships between a polyline  $L$  and a polygon  $R$  is based on the nine intersections of  $L$ 's interior, boundary, and exterior with the interior, boundary, and exterior of  $R$ . A  $3 \times 3$  matrix  $M$ , called the 9-intersection, concisely represents these criteria.

$$M = \begin{pmatrix} L^O \cap R^O & L^O \cap \partial R & L^O \cap R^- \\ \partial L \cap R^O & \partial L \cap \partial R & \partial L \cap R^- \\ L^- \cap R^O & L^- \cap \partial R & L^- \cap R^- \end{pmatrix}$$

Starting from these studies, two other models of conceptual similarity among topological relations between a polyline and a region were developed [8]. A further study of spatial similarity and a computational method to evaluate the similarity of spatial scenes based on the ordering of spatial relations is discussed in [9]. More recently, two papers have studied the spatial neighbourhoods between objects [11, 12]. In [11] the authors studied topological relationships between two regions, comparing two strategies for minimizing topological constraints in a query expressed by a visual example, and giving search results in terms of number and similarity values. In [12] the authors present an idea on how qualitative spatial reasoning can be exploited in reasoning on action and change. They investigate how its conceptual neighbourhood structure can be applied to the situation calculus for qualitative reasoning on relative positional information.

The paper is structured as follows: in Section 2 the GeoPQL operators are briefly introduced and two query examples are also shown. Section 3 proposes a computational model to determine the most conceptually similar relationships for each configuration, and the three cases polygon-polygon, polyline-polygon and polyline-polyline are studied. Section 4 gives an example of transformation to an approximate query, and finally Section 5 concludes.

## 2. THE GEOPQL OPERATORS

The GeoPQL algebra consists of 12 operators: Geo-union (UNI), Geo-difference (DIF), Geo-disjunction (DSJ), Geo-touching (TCH), Geo-inclusion (INC), Geo-crossing (CRS), Geo-pass-through (PTH), Geo-overlapping (OVL), Geo-equality (EQL), Geo-distance (DIS), Geo-any (ANY), and Geo-alias (ALS). Geo-touching refers to a pair of touching graphical objects, Geo-crossing refers to the crossover between two polylines, Geo-pass-through refers to a polyline which passes through a polygon, Geo-alias allows an SGO to be duplicated in order to express the OR operator, and Geo-any allows any relationship between a pair of SGO to be considered valid, i.e., no constraint exists between them. This last operator allows an unambiguous visual query to be obtained, as explained in [4].

## 3. THE COMPUTATIONAL MODEL

The answer to a query may sometimes be “zero elements”. In this situation, it would be useful if the system automatically relaxed one or more topological constraints until a positive result is achieved. To do this, we need to define the *operator conceptual similarity (OCS) graph*. This is obtained from the configuration of two disjointed (DSJ) objects through three operations: object shifting, rotation, and size change (smaller, larger). If we consider two disjointed polygons as our initial configuration, as shown in Figure 1(a), their relative positions can

be modified by shifting and/or rotating one of them. By shifting (or rotating and shifting) polygon B towards A, we obtain the second situation, in which they are touching (Figures 1(b) and 1(c)). Note that polygon B can touch polygon A in two different ways: at one point or along a line.

If we continue to shift polygon B we obtain the third situation, which presents in reality two sub-situations. The former (Figures 1(d) and 1(f)) produces only an overlap between the two objects, while the latter (Figure 1(e)) produces an overlap and a touch. Still shifting polygon B, we pass from Figure 1(e) (OVL + TCH) to Figure 1(f) (OVL), or to Figure 1(h) (INC + TCH).

From Figure 1(h) we pass to Figure 1(i) (INC), or, with a shift towards the outside, to Figure 1(f) (OVL).

The enlargement of SGO B as shown in Figure 1(j) causes it to coincide with SGO A (Figure 1(k)). But it is also possible to move from the condition “TCH + INC”, shown in Figures 1(g) and 1(h), to the condition “EQL”, shown in Figure 1(k). In contrast, by shifting SGO B as shown in Figure 1(h), we obtain the condition shown in Figure 1(i) (INC). Finally, from Figure 1(g) we can also obtain the condition shown in Figure 1(j) (INC).

We have six possible conceptual similarity graphs which refer to all the combinations between two features (point-point, point-polyline, etc.). However, the three cases in which one point is one of the two SGO are sub-cases of the others. We will therefore study only three cases: polyline-polyline, polygon-polygon and polygon-polyline.

Not all the operators defined in GeoPQL are considered in each case. For example, in the polyline-polyline graph, the operator Pass-through (PTH) does not need to be considered, as it is valid only in the case of polygon-polyline. In the polygon-polyline graph, the operator Cross (CRS) need not be considered as it is valid only in the polyline-polyline case. Similarly, Overlap (OVL) is valid only in the case of polygon-polygon. We now consider and discuss the three cases separately.

### 3.1 Case of Two Polygons

The graph shown in Figure 2(a) represents the *OCS graph* for the pair “polygon-polygon”. It begins from the condition of two disjointed SGOs (row of the graph). Using one of the three operations described above to modify the configuration, we arrive, step by step, at the leaf of the graph, i.e. the operator EQL. The 4x4 matrix structure that enlarges the classic 9-intersection matrix [8] is shown in Figure 2(b). In the 9-intersection matrix the authors consider an object’s internal, boundary and external points and their position with respect to the other object (without distinguishing if the boundary contact points consist of one or more points or lines and without considering the number of times that each condition is verified). In contrast, in the 4x4 matrix of Figure 2(b) we distinguish the kind of boundary between the two SGO (point and/or line), as well as the number of times which each condition can arise (expressed as a whole, positive number inside the crossover point in the matrix of the points considered for each SGO). Each side, representing in this case one polygon, has three rows, which refer to the points of each polygon: i (interior), b (boundary), and e (exterior). The boundary points are subdivided into p (point) and l (polyline). The values inside the matrix represent the number of times that a given configuration appears in the sketch representing the pictorial query. In Figure 2(b), for example, the symbol “-” represents an impossible case and n is a generic natural number.

Figure 1

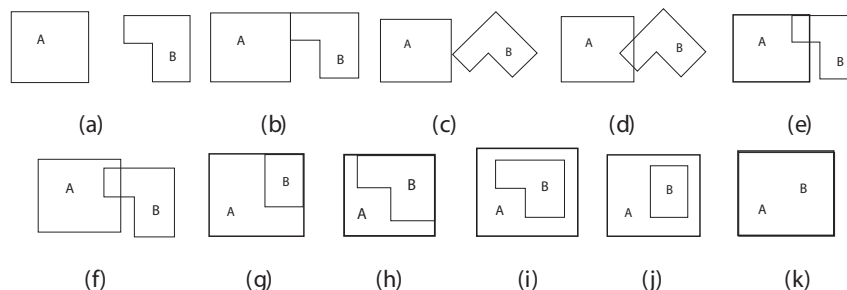
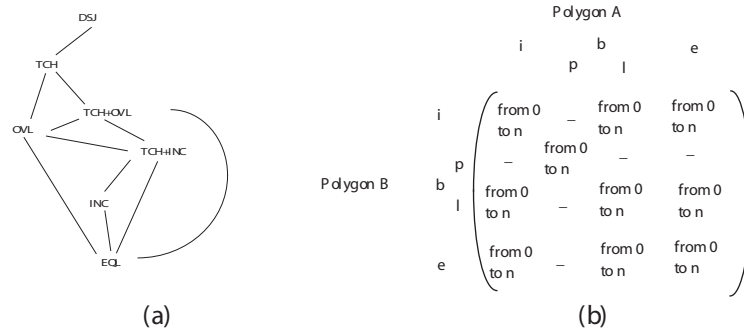


Figure 2



### 3.2 Case of Polygon-Polyline

The *OCS graph* of the operators valid for the polygon-polyline case is shown in Figure 3(a). The graph was obtained from the configuration of two disjointed (DSJ) objects. The three operations described above are applied to the polyline (or polygon) to obtain all other possible configurations (from touching to pass-through, etc.). The 4x4 matrix structure, which enlarges the classic 9-intersection matrix, is shown in figure 3(b).

### 3.3 Case of Two Polylines

The *OCS graph* of the operators valid for the polyline-polyline case is shown in Figure 4(a). Again, the graph was obtained from the disjointed polyline-polyline configuration (DSJ), by shifting, rotation, and extending/shortening.

The matrix structure for this case (a 5 x 5 matrix), which enlarges the classic 9-intersection matrix, is shown in Figure 4(b), while the part within the two lines is explained (in its different configurations) in Figure 4(c).

Here too, we can obtain the three different configurations TCH, EQL, and INC from two disjointed polylines, as shown in Figure 5(1(a)), 5(1(b)), and 5(1(c)).

Continuing this procedure, we can obtain four other configurations from the configuration of Figure 5(2(a)).

With a rotation of  $B_1$  we obtain Figure 5(2(b)) (EQL, equal) if  $B_1 = A$  in length, or Figure 5(2(c)) (INC) if  $B_1$  is shorter than  $A$ ; if the two lines touch (TCH) as in Figure 5(3(a)), shifting  $B$  results in the crossover (CRS) configuration of Figure 5(3(b)). Finally, if the starting configuration is that shown in Figure 5(3(c)), by lengthening  $B$  we obtain Figure 5(3(d)) (CRS + TCH).

In Figure 5(2(c)), lengthening  $B$  results in Figure 5(2(b)) (EQL), while, from Figure 5(3(b)) we obtain Figure 5(2(c)) (INC), if  $B$  is shorter than  $A$ , or Figure 5(2(b)) (EQL) if  $B = A$ .

From the configuration of Figure 5(2(d)) we can then obtain Figure 5(2(e)) (CRS), similar to Figure 5(2(b)).

If  $A = B$  in length, rotating  $B$  in Figure 5(2(b)) results in Figure 5(1(b)) (EQL).

The exterior points are subdivided into two:  $x$  and  $\bar{x}$ , which represent the two semi-planes obtained by extending one polyline from the two points of boundary. The terms  $X_1$ ,  $X_2$ ,  $Y_1$  and  $Y_2$  assume the numeric value 0 or 1 or 2 depending on the value of  $b / \bar{b}$  (boundary of the Line A and boundary of the Line B), as shown in Figure 4(c).

If the boundary of line A is not in common with that of line  $B_1$  they (two points) may be all in the semi-plane  $x$  of B, or all in the semi-plane  $\bar{x}$  of B, or one in  $x$  and one in  $\bar{x}$ . So, in correspondence with "0" of  $b$  (A) cross  $\bar{b}$  (B) we have 0 and 2, 2 and 0 or 1 and 1, in correspondence with  $B$  (e ( $x$  and  $\bar{x}$ )), as shown in Figure 4(c).

If the line A boundary has a point in common with a point of the line B boundary, the other boundary point is in either the semi-plane  $x$  of B or the semi-plane  $\bar{x}$  of B. So, in correspondence with "1" of  $b$  (A) cross  $\bar{b}$  (B) we have either 1 and 0 or 0 and 1, in correspondence with  $B$  (e ( $x$  and  $\bar{x}$ )), as shown in Figure 5(4). Finally, if both points of the line A boundary are in common with both points of the line B boundary then in correspondence with "2" of  $b$  (A) cross  $\bar{b}$  (B) we have 0 and 0, in correspondence with  $B$  (e ( $x$  and  $\bar{x}$ )), as shown in Figure 5(5).

Figure 3

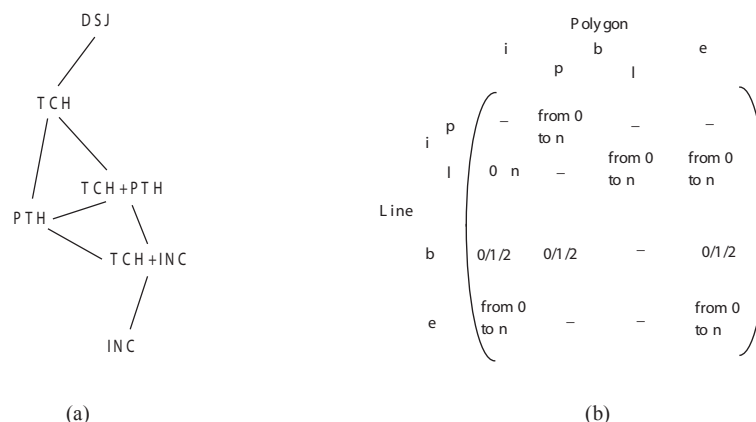


Figure 4

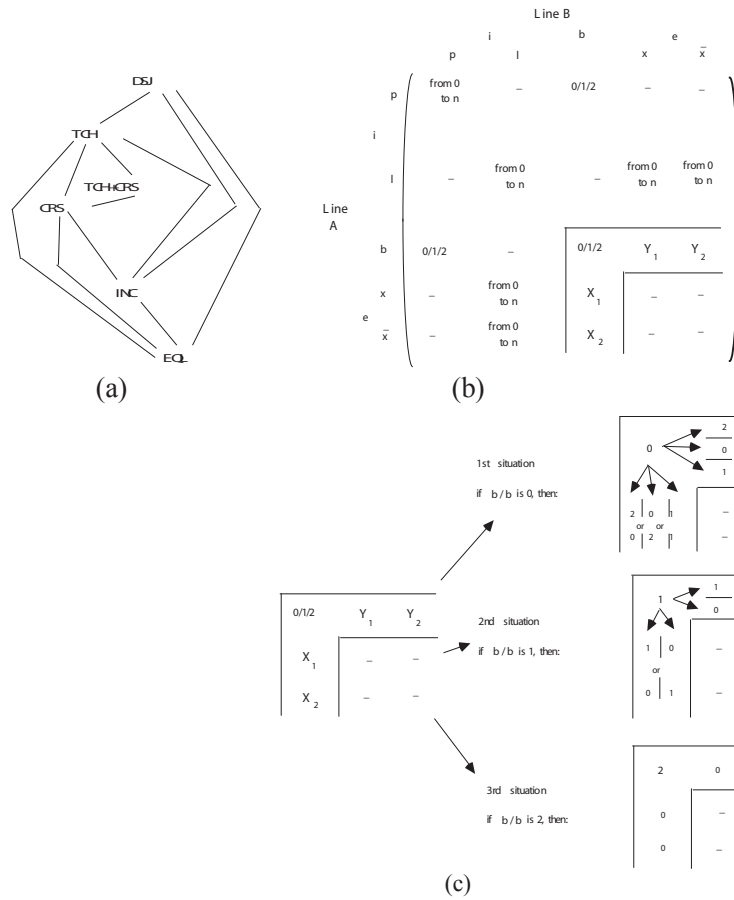


Figure 5

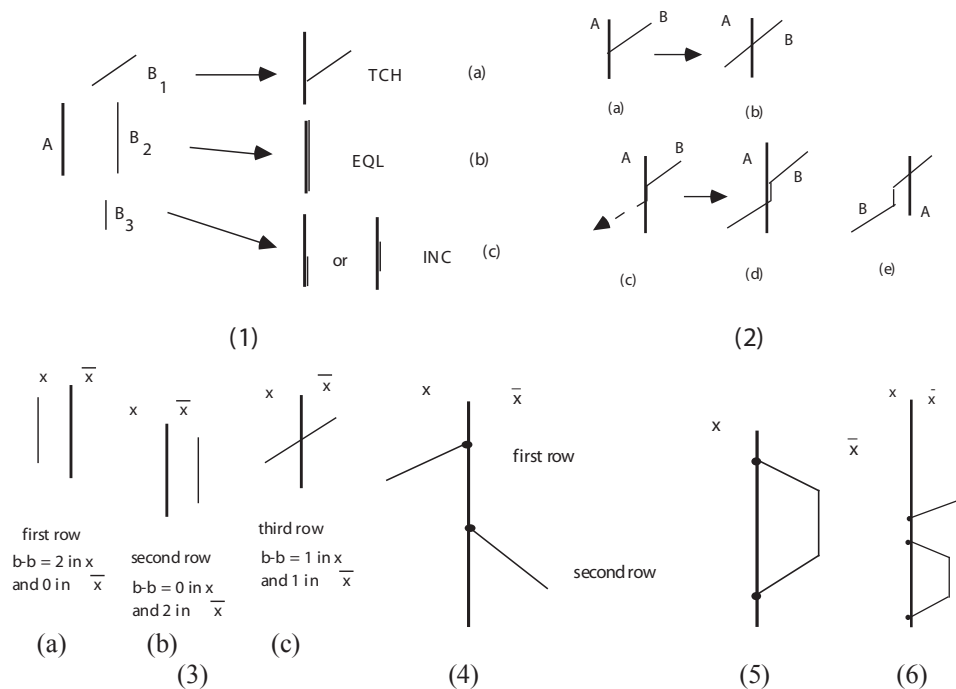
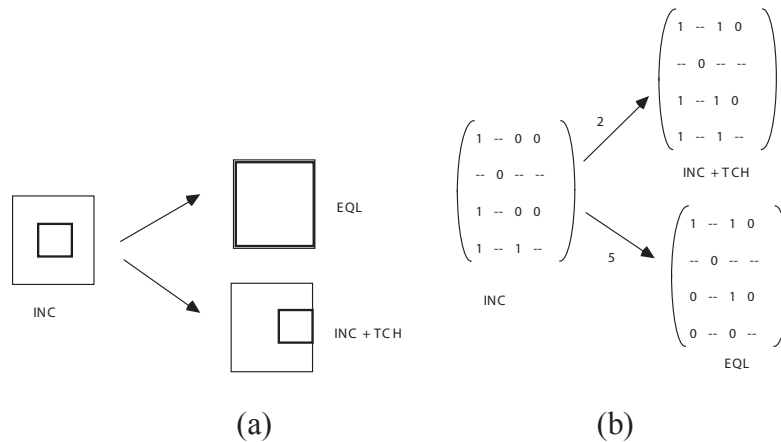


Figure 6



Obviously, the boundary points of one line can be 0, 1 or 2 internal points of the other, as shown in Figure 5(6).

#### 4. APPROXIMATE QUERIES RELAXING OPERATORS

Suppose that the user formulates the query:

*“Find all the lakes which are Inside a Province”*

Using GeoPQL, the pictorial query drawn is shown in Figure 6(a). Suppose also that the query gives “zero elements” as the answer.

The system then asks if the user wishes to relax the topological constraint “INC” and, if “yes”, it goes to the neighbourhoods relative to “two polygons”, selects the operator INC and determines the nearest configurations: in this case, INC+TCH (by a shift) and EQL (by an extending), as shown in Figure 6(b). For each of these pairs the *degree of neighbourhoods*, obtained from the matrix of Figure, is evaluated. By “degree of neighbourhoods” we mean the number of changes in the new matrix compared with the original (from the INC to the INC+TCH matrix and from the INC to the EQL matrix).

In the first case, we have “2” (the third value of the first row, and the third value of the third row). We leave the reader to evaluate the second case.

The minor degree of neighbourhoods (also called *weight*) between the two relaxations allows the system to automatically select the relative operator(s) (in our case, INC+TCH) by which the original operator (in our case, INC) is relaxed. Following the original, unsuccessful query, the system therefore automatically processes the query “Find all the lakes which are **Inside Or Touch** a Province”. The new result is still evaluated.

The procedure stops when a result is found.

#### 5. DISCUSSION AND CONCLUSION

In this paper we proposed a complex matrix for each of the three configurations polygon-polygon, polygon-polyline, and polyline-polyline and a computational model for the similarity of their spatial relationships. This is described by an *operator conceptual similarity (OCS) graph*, linking the most similar relationships and defining the weight of each relaxation.

A large number of different configurations exist between a polyline and a polygon, two polylines and two polygons - more than the 19 binary topological relationships presented in [1, 2]. We have defined a correspondence between these configurations and said 19 topological relationships and have considered the number of contact points (whether touching or crossing) between the two objects, i.e. the cardinality (points number) of the intersection between the polyline interior and

the polygon boundary. These considerations led us to consider a more complex matrix and the relative OCS *graph* for each pair of objects in order to design a computational model to determine more conceptually similar relationships for each configuration. We discussed the three different configurations and gave an example of query relaxation.

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