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Exotic Options with Stochastic Volatilities

Sanju Vaidya, Division of Math & Computer Information Science, Mercy College, Dobbs Ferry, NY 10522

SECTION 1. INTRODUCTION

In 1973, Fischer Black and Myron Scholes made a major breakthrough by developing a model for pricing stock options. In 1997, they were awarded the Nobel Prize in Economics for their outstanding work. The Black-Scholes model and its extensions are very popular for pricing many types of options. Individuals, corporations, and many financial institutions use derivatives like options and futures to reduce risk exposures.

The Black-Scholes model is based on the assumption that the asset volatility is either constant or a known function of time over the life of the option. In 1987, J. Hull and A. White examined the problem of pricing of a European call option on a stock that has a stochastic volatility. They proved that when there is a positive correlation between the stock price and its volatility, out-of-the money options are under priced by the Black-Scholes formula, while in-the-money options are overpriced. When the correlation is negative, the effect is reversed.

Derivatives with more complicated payoffs than the standard European or American call options and put options are sometimes referred to as exotic options. Most exotic options trade in over-the counter market and are designed by financial institutions to meet the requirements of their clients.

In Vaidya [6], some computational techniques for pricing standard options and certain exotic options, Lookback options are discussed. The payoff from the Lookback options depend upon the maximum or minimum stock price reached during the life of the option. In 1979, M. Goldman, H. Sosin, and M. Gatto found valuation formulas for European Lookback Call and put options on a stock when the volatility is constant or a known function of time.

In Vaidya [7], the problem of pricing European Lookback call options on a non-dividend paying stock with stochastic volatility is examined. It turned out that the price of a European lookback call on a stock that has a stochastic volatility has a bias relative to the price of the European lookback call on the stock with constant volatility. When the volatility is positively correlated with the stock price, the price of the European lookback call options is below the price of European lookback call options on the stock with constant volatility. When the volatility is negatively correlated with the stock price, the reverse is true.

Another important type of Exotic options is Asian options. The payoff from an Asian option depends upon the average price of the underlying asset during the life of the option. There are no exact formulas for pricing Asian options. In 1993, J. Hull and A. White found efficient procedures for pricing these average options on stocks when the volatility is constant. So this leads to the following question.

Research Question: *What happens to pricing of European average call options on assets when the volatility is stochastic?*

Research Method and Conclusion: *We will use Monte Carlo Simulation to analyze the problem. We will use the antithetic variable technique and the control variable technique as described in Hammersley and Handscomb [3] and J. Hull and A. White [4].*

It turned out that the price of a European average call option on a stock that has a stochastic volatility has a bias relative to the price of the European average call on the stock with constant volatility. When the volatility is positively correlated with the stock price the price of European average call options is below the price of the options on the stock with constant volatility. When the volatility is negatively correlated with the stock price the price of European average call options is above the price of the options on the stock with constant volatility.

SECTION 2. NOTATION AND TERMINOLOGY

We will use the notation and terminology from Sections I and II of Hull and White [4].

SECTION 3. RESEARCH METHOD AND CONCLUSION

We will use Monte Carlo Simulation to analyze the problem. We will use the antithetic variable technique and the control variable technique as described in Hammersley and Handscomb [3] and Hull and White [4]. The time interval *(T - t)* is divided into *n* equal subintervals. Two independent standard normal variates x_i and y_i (where i is from 1 to n) are sampled. They are used to generate the stock price S_i and variance V_i at time *i* in a risk neutral world using the following formulas.

$$S_i = S_{i-1} e^{\left[\left(r - V_{i-1}/2\right)\Delta t + x_i \sqrt{V_{i-1}\Delta t}\right]}$$

$$V_i = V_{i-1} e^{\left[\left(\mu - \zeta^2/2\right)\Delta t + \rho x_i \zeta \sqrt{\Delta t} + \sqrt{1-\rho^2}\, y_i \zeta \sqrt{\Delta t}\right]}$$

Let X be the strike price and S_{avg} be the arithmetic average of the stock prices $S_0, S_1, ...S_n$. The value of $e^{-r(T-t)}\left[\max(0, S_{avg} - X)\right]$ is calculated to give one sample value p_1 of option price. A second price p_2 is calculated by replacing x_i with $-x_i\left(1 \le i \le n\right)$ and repeating the calculations p_3 is calculated by replacing y_i with $-y_i(1 \le i \le n)$ and p_4 is calculated by replacing x_i with $-x_i$ and y_i with $-y_i(1 \le i \le n)$. Then two samples values q_1 and q_2 are calculated by simulating S using x_i with $-x_i$ with V kept constant at V_0. This gives the following two estimates of the pricing bias: $\frac{p_1 + p_3 - 2q_1}{2}$ and $\frac{p_2 + p_4 - 2q_2}{2}$. These estimates are averaged over a larger number of simulations.

It turned out that the price of a European average call option on a stock that has a stochastic volatility has a bias relative to the price of the European average call on the stock with constant volatility. When the volatility is positively correlated with the stock price the price of European average call options is below the price of the options on the stock with constant volatility. When the volatility is negatively correlated with the stock price the price of European average call

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options is above the price of the options on the stock with constant volatility.

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