One Simple Method for Design of Multiplierless FIR Filters

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ABSTRACT
This paper presents one simple method for a design of multiplierless finite impulse response (FIR) filters by a repeated use of the same filter. The prototype filter is a cascade of a second order recursive running sum (RRS) filter and its corresponding expanded versions. As a result, no multipliers are required to implement this filter.

1. INTRODUCTION
Linear phase finite impulse response (FIR) filters of length \( N \) require \((N+1)/2\) multipliers, \( N-1 \) adders and \( N-1 \) delays. The complexity of the implementation increases with the increase in the number of multipliers. Over the past years there has been a number of attempts to reduce the number of multipliers like Adams and Willson, 1983, Adams and Wilson 1984, Ramakrishnan, 1989, Bartolo at all., 1998, etc. Another approach is a true multiplier-less design where the coefficients are reduced to simple integers or to simple combinations of powers of two, for example, Tai and Lin, 1992, Yli-Kaakinen and Saramaki, 2001, Maw-Ching Liu at all, 2001, Coleman, 2002, etc.

In this paper we propose to use the sharpening technique of Kaiser and Hamming, 1977, where the performance of a symmetric FIR filter can be improved by a multiple use of the same filter, based on the Amplitude Change Function (ACF).

The polynomial relationship between the amplitudes \( H \) and \( H_0 \) of the prototype and of the transformed filters can be expressed as

\[
H_0 = H^{n+1} \sum_{k=0}^{m} C(n+k,k)(1-H)^k,
\]

where \( C(n+k,k)=(n+k)!/n!k! \) denotes the binomial coefficient, and \( n \) and \( m \) are the tangency orders at \( H=0 \) and \( H=1 \), respectively. The improvement in the passband, near \( H=1 \), or in the stopband near \( H=0 \) depends on the order of tangency of the ACF at \( H=1 \) or at \( H=0 \), respectively.

The prototype filter must be simple in order to avoid computational complexity. The prototype filter chosen here is a cascade of the second order recursive running sum (RRS) filter and the expanded second order RRS filters. In that way, all coefficients of the prototype filter have value 1 and therefore no multiplication is needed. The system function of the second order RRS filter and its expanded version, expanded by interpolation factor \( M_k \), are given by the following relations:

\[
G(z) = (1+z^{-1})/2
\]

\[
G(z^{M_k}) = (1+z^{-M_k})/2, \quad M_k = 2,3,4,\ldots \tag{2}
\]

The corresponding magnitude responses are given by

\[
G(e^{j\omega}) = \cos(\omega/2)
\]

\[
G(e^{j\omega M_k}) = \cos(\omega M_k/2) \tag{3}
\]

The cascade of \( K \) filters (3) has the following magnitude response:

\[
G(e^{j\omega}) = \cos(\omega / 2) \cos(\omega 2 / 2) \cos(3\omega / 2) \cdots \cos(K\omega / 2). \tag{4}
\]

As an illustrative example, the magnitude response for the case \( K=5 \) is shown in Figure 1.

We can notice that the magnitude characteristic has low stopband attenuation and the high passband droop. In order to improve the magnitude characteristic we use the ACF function as explained in the next section.

2. DESIGN PROCEDURE
We consider a typical lowpass filter with the passband edge \( w_p \) and the stopband edge \( w_s \).

The stopband frequency determines the number \( K \) as follows

\[
K = \frac{2w_s}{\pi}
\]

Figure 1: Magnitude response of the prototype filter for \( K=5 \).
where \( \text{int}\{\}\) means the closest integer of \{\}. The main idea is to, instead of applying the corresponding ACF to the entire cascade, apply different ACF’s to a different filters of the cascade (4). The reason is because the filters in the cascade (4) with higher value \( M \) have more droop in the passband and so they need the ACF with higher values of \( m \) and \( n \). At the other extreme, the filters with smaller values of \( M \) have wider passband and consequently need lower values of \( m \) and \( n \) in the ACF. We divide the cascade of filters into the subgroups and apply a different ACF to each subgroup. The number of filters in a subgroup is at most 3. As a result, the highest values for \( m \) and \( n \) are typically 3. The procedure is implemented in MATLAB.

The following example illustrates the method.

**Example 1:**
We design a filter with these specifications: The passband and stopband frequencies are \( w_p = 0.42 \) and \( w_s = 0.146 \), respectively. The passband ripple is \( R_p = 0.2 \) dB and the stopband attenuation is \( A_s = -35 \) dB. A design using the Parks McClellan algorithm results in a filter of the order 62 and requires 31 multipliers. From (5) it follows that \( K = 7 \).

Therefore the prototype filter is

\[
H(e^{j\omega}) = \cos(\omega/2) \cos(3\omega/2) \cos(7\omega/2). 
\]

We first form the following groups:
\[
G_1 = \cos(7\omega/2) \cos(6\omega/2), \\
G_2 = \cos(5\omega/2) \cos(4\omega/2) \cos(3\omega/2), \\
G_3 = \cos(\omega). 
\]

**Figure 2: Example 1**

(a) Magnitude response

(b) Passband zoom

(c) Stopband zoom

Next we apply these ACF functions, \( H_i(G_j, m, n), i = 1, \ldots, 4 \).

\[
\begin{align*}
H_1(G_1,3,3) &= 35G_1^4 - 84G_1^5 + 70G_1^6 - 20G_1^7, \\
H_2(G_2,3,2) &= 20G_2^3 - 45G_2^4 + 36G_2^5 - 10G_2^6, \\
H_3(G_3,1,1) &= -2G_3^1 + 3G_3^2, \\
H_4(G_4,1,1) &= 2G_4^2 - 3G_4^3. 
\end{align*}
\]

The results of this design are shown in Figure 2. Figure 2 (a) shows the magnitude response. The passband zoom in Figure 2 (b) demonstrates that the passband specification is satisfied, while the stopband in Figure 2 (b) shows that the stopband specification is also satisfied.

**3. CONCLUSIONS**
A simple method for the design of multiplierless FIR filters is presented. The method uses the cascade of a second order RRS filter with the corresponding expanded filters. The number of filters in the cascade depends on the order of the stopband frequency. In contrast to the method proposed in Tai and Lin, 1992, where the orders of tangencies are varied from 1 to 8, in the method proposed here the orders of tangencies are varied only from 1 to 3, thereby resulting in a less complex filter.

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