IDEA GROUP PUBLISHING

701 E. Chocolate Avenue, Suite 200, Hershey PA 17033, USA
Tel: 717/533-8845; Fax 717/533-8661; URL-http://www.idea-group.com

ITP4704

Computational Techniques for Pricing Options

Dr. Sanju Vaidya
Division of Math & Computer Information Science, Mercy College
555 Broadway, Dobbs Ferry, NY 10522, USA
Phone: (914) 674-7536, Fax: (914) 674-7518
Email: svaidya@mercy.edu

SECTION 1: INTRODUCTION

In 1973, Fischer Black and Myron Scholes [1] made a major breakthrough by developing a model for pricing stock options. In 1997, they were awarded the Nobel Prize in Economics for their outstanding work. The Black-Scholes model and its extensions are very popular for pricing many types of options. Individuals, corporations, and many financial institutions use derivatives like options and futures to reduce risk exposures.

The Black-Scholes model is based on the assumption that trading takes place continuously in time. In 1979, J. Cox, S. Ross, and M. Rubinstein [4] developed a binomial model that is based on discrete trading time. In applications such as American option valuation, the binomial model is widely used by many financial institutions.

Derivatives with more complicated payoffs than the standard European or American call options and put options are sometimes referred to as exotic options. Most exotic options trade in over-the counter market and are designed by financial institutions to meet the requirements of their clients.

In this paper we will discuss some computational techniques for pricing standard options and certain exotic options, American Lookback put options. The payoff from the Lookback options depend upon the maximum or minimum stock price reached during the life of the option. In 1992, S. Babbs [2] used the binomial tree approach for pricing American Lookback put options. Prof. E. Reiner proposed the same approach in a lecture at Berkeley.

The tree approach has many advantages. It could be used for both European and American style options. When exact formulas are not available (e.g. American put option), numerical procedures such as Monte Carlo simulation, binomial and trinomial trees are widely used in real life. Moreover, the analytic results assume that the stock price is observed continuously. But if the stock price is observed in a discrete manner, say once a day, to calculate the maximum or the minimum, the tree approach makes more sense.

Research Question

The main problem of the tree approach is the convergence of the values, which is slow. A large number of time steps are required to obtain a reasonably accurate result. So the question is as follows: How can we increase the speed of the algorithm? Can we reduce the computation time and get accurate results?

Research Method and Conclusion

In this paper, we will show that if we use Hull's Control Variate technique as described in [6], then the convergence of the values is much faster and the tree approach for the American Lookback put options is more efficient. By using Hull's Control Variate technique, we could reduce the number of arithmetical calculations performed by the algorithm. So the computation time is reduced and we get accurate results. As we know, saving in computation time is very important in a trading room where thousands of derivative prices need to be updated regularly.

SECTION 2: NOTATION AND TERMINOLOGY

In this section we will introduce the notation and terminology, which will be used throughout the paper. First let us define some standard concepts in option pricing.

There are two basic types of options. A **call option** gives the holder the right to buy the underlying asset by a certain date for a certain price. **A put option** gives the holder the right to sell the underlying asset by a certain date for a certain price. The price in the contract is known as the **exercise price or strike price**; the date in the contract is known as the **expiration date, exercise date, or maturity. American options** can be exercised at any time up to the expiration date. **European options** can be exercised only on the expiration date itself.

Derivatives with more complicated payoffs than the standard European or American calls or puts are referred to as **exotic options**. The payoff from a **European-style lookback put** is the amount by which the maximum stock price achieved during the life of the option exceeds the final stock price. When the **American-style lookback put** is exercised, it provides a payoff equal to the excess of the maximum stock price over the current stock price. We will use the following notation:

S: current stock price
X: Strike price of option
T: time of expiration of option
t: current time
r: risk-free rate of interest for maturity T (continuously compounded)
σ : volatility of stock price

SECTION 3. REVIEW OF PRICING OPTIONS

In this section, we will review some models for pricing options.

Black-Scholes Model: In 1973, Fisher Black and Myron Scholes [1] developed a model for pricing European call and put options. The exact formulas are as follows.

$$c = SN(d_1) - Xe^{-r(T-t)}N(d_2) \text{ and}$$

$$p = Xe^{-r(T-t)}N(-d_2) - SN(-d_1)$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

c = value of the European call option
p = value of the European put option

There is no exact formula for the value of an American put option on a non-dividend paying stock. When exact formulas are not available, numerical procedures such as Monte Carlo Simulation, Finite Difference Methods, Binomial and Trinomial trees are used to value derivatives.

Binomial Model of Cox, Ross, and Rubinstein: In 1979 C.Cox, S Ross, and M.Rubinstein [4] developed a binomial model for pricing options.

They used the binomial tree approach. This is a tree that represents possible paths that might be followed by the underlying stock price over the life of the option. The price of an option is calculated by discounting the expected payoff from the option.

The idea is as follows: - First divide the life of the option into small time intervals of length Δt. Assume that in each time interval the stock price moves from its initial value of S to one of two new values, Su and Sd; u is an up factor (u > 1), and d = 1/u which is the down factor. So at time Δt there are two possible stock prices, Su and Sd; at time 2 Δt there are three possible stock prices Su^2, S, and Sd^2 and so on. Continuing in this manner, we will get a binomial tree. The value of the option is known at the end of the tree (time T). For example, a put option is worth max $(X - S_T, 0)$, where S_T is the stock price at time T and X is the strike price. Then the value at each node at time T- Δt can be calculated as the expected value at time T discounted at rate r for a time period Δt. Similarly, the value at each node at time T – 2 Δt can be calculated, and so on. Finally, working backward, the value of the option at time zero (current time) is obtained.

SECTION 4: VALUATION OF AMERICAN LOOKBACK PUT OPTIONS

The payoff from the Lookback options depend upon the maximum or minimum stock price reached during the life of the option. In 1992, S. Babbs [2] used the binomial tree approach for pricing American Lookback put options. The idea is as follows: - Let F(t) be the maximum stock price achieved up to time t and S(t) be the stock price at time t. Let Y = F(t)/S(t). Then we have to construct a binomial tree for Y. Initially, Y is 1 since F = S at time zero. If there is an up movement in S during the first time step, then Y = 1. If there is a down movement in S during the first time step, then Y = 1/d = u. Continuing in this way we have a binomial tree for Y. Then we have to roll back through the tree to find the value of the option. The approach calculates the option price as the discounted value of the expected option pay-off.

Research Question

The main problem of the tree approach is the convergence of the values, which is slow. A large number of time steps are required to obtain a reasonably accurate result. So the question is as follows: How can we increase the speed of the algorithm? Can we reduce the number of time steps and get accurate results?

SECTION 5: RESEARCH METHOD AND CONCLUSION:

In this section, we will show that if we use the Control Variate technique as described in the paper [6] by Hull and White, then the convergence of the values is much faster and the tree approach for the American Lookback put options is more efficient.

Control Variate technique

The control variate technique can be used to improve the efficiency of numerical valuation procedures. It is applicable when we wish to value an option A, and we have an accurate solution for a similar option, B. Boyle suggested the use of control variate technique in conjunction with Monte Carlo simulation. Hull and White used it for lattice approach. The key element in the technique is that the same numerical procedure is used to value both option A, and option B even the accurate value for option B is known. To explain the technique, let V(A) = value of the option A to be determined, V(B) = accurate value of option B, V*(A) = estimated value of option A from the numerical procedure, and V*(B) = estimated value of option B from the numerical procedure. Then V(A) = V(B) + (V*(A) – V*(B)) gives a better estimate for the value of option A.

How to use the Control Variate technique to reduce the computation time for American lookback put option?

Let A be the American lookback put option on a non-dividend paying stock S and B be the European lookback put option on the same stock, with the same strike price and the maturity date. Goldman, Sosin, and Gatto [5] found valuation formulas for European lookback call and put options. So using their formula for put option we have the accurate value of option B. The formula is as follows.

$$S_{\max} e^{-rT}\left(N(b_1) - \tfrac{\sigma^2}{2r} e^{Y} N(-b_3)\right) + S\,\tfrac{\sigma^2}{2r} N(-b_2) - SN(b_2)$$

$$\textit{where}$$

$$b_1 = \frac{\ln(S_{\max}/S) + (-r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$b_2 = b_1 - \sigma\sqrt{T}$$

$$b_3 = \frac{\ln(S_{\max}/S) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$Y = \frac{2(r - \frac{\sigma^2}{2})\ln(S_{\max}/S)}{\sigma^2}$$

$S_{\max}$ is the maximum stock price achieved to date.

As described in Section 4, we could use the binomial tree for both the option A and B and find their estimated values. So by the Control Variate technique as described earlier, we have V(A) = V(B) + (V*(A) – V*(B)).

It turns out that the Control Variate technique improves the efficiency of the binomial tree approach suggested by S. Babbs for American lookback put options.

BIBLIOGRAPHY

1. F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, 81 (May-June 1973), 637-659.
2. S. Babbs, "Binomial Valuation of Lookback Options", Working paper, *Midland Global Markets*, 1992.
3. P. Boyle, "Options; a Monte Carlo Approach," Journal of Financial Economics, 4 (May 1977), 323- 338.
4. J. Cox, S. Ross, and M. Rubinstein, "Option Pricing: A Simplified Approach", *Journal of Financial Economics*, 7 (October 1979), 229-264.
5. B. Goldman, H. Sosin, and M. A. Gatto, "Path-dependent Options: Buy at the Low, Sell at the High," Journal of Finance, 34 (December 1979), 1111-27.
6. J. Hull and A. White, "The Use of Control Variate Technique in Option Pricing", *Journal of Financial and Quantitative Analysis*, 23 (September 1988), 237–251.
7. J. Hull, Options, Futures, and Other Derivatives, third edition, Prentice Hall, 1997.

Related Content