Fuzzy Complex System of Linear Equations

F

Gizem Temelcan
https://orcid.org/0000-0002-1885-0674
Beykoz University, Turkey

Hale Gonce Köçken
https://orcid.org/0000-0003-1121-7099
Yildiz Technical University, Turkey

İnci Albayrak
Yildiz Technical University, Turkey

INTRODUCTION

The widespread use of linear assumptions in engineering and science requires that many studies begin by establishing a linear model. Also, most advanced problems require solving linear systems with real or complex parameters, often of very large dimensions.

In machine learning, a machine is trained to learn a concept by building models to distinguish classes of objects. It would be appropriate to consider the fuzzy logic approach when there is no definite line separating the two classes, or when the distinguishing features are defined indistinctly. Thus, in machine learning algorithms, the application of uncertainty modeling and decision-making methods leads to better performances of algorithm behavior. With this understanding, the contribution of modeling fuzzy complex systems of linear equations to algorithm behavior will be important. For instance, drawing a scatter plot can be given in data science. A scatter plot is drawn via dots to represent values for two different variables. The position of each dot on the horizontal and vertical axis indicates values for an individual data point. Scatter plots are used to observe relationships between variables. To find the best fitting line, linear regression, which is an example of SLE, can be used. Thus, to predict numerical values, linear regression is used in machine learning. Moreover, Support Vector Machine (SVM) learning algorithms are used to build accurate models with practical relevance for classification, regression, and novelty detection. Some applications of SVMs are facial recognition, text categorization, and bioinformatics. Generally, the training task requires solving an SLE in SVM learning algorithms (Do & Fekete, 2007). Furthermore, in deep learning, SLE is used when training a deep model. For more applications, see (Dombi & Kertesz-Farkas, 2009; Gan, 2013; Martínez et al., 2015; Kurnianggoro et al., 2015; Gan & Huang, 2017; Saini et al., 2020; Jo, 2021).

Systems of linear equations (SLE) play an important role in the areas such as mathematics, statistics, economics, physics, chemistry, social sciences, and engineering. They have many application areas in physical and engineering sciences such as circuit analysis, structural mechanics, heat transport, fluid flow, etc. For instance, civil engineers use the systems to design and analyze load-bearing structures such as bridges; mechanical engineers to design and analyze suspension systems, and electrical engineers to design and analyze electrical circuits. A standard real SLE can be written as $AX = b$, where A and b are crisp real matrices, i.e., parameters, and X is the real variable vector. SLE has a wide range of studies in the literature. There are many methods for solving these systems having crisp real parameters

DOI: 10.4018/978-1-7998-9220-5.ch118

and variables. Several authors have proposed direct and iterative methods for solving the SLE. Direct methods such as Gaussian elimination (with/without pivot technique) and LU decomposition produce the exact solution assuming no rounding errors. On the other hand, iterative methods such as Jacobi, Seidel-Gauss methods, Conjugate gradient method, sequential over relaxation technique produce an approximate solution whose accuracy is imposed by the user. These procedures can be presented as two major classes of methods for the numerical solution of SLE with real coefficients. In the case of a large linear system, the direct solution of a sparse matrix linear system can be obtained by direct methods, without ignoring the computational cost and the corresponding total time. Iterative methods, whose efficiency depends on the method chosen, represent a better alternative.

A concept or information, even if vague and imprecise, may not be applicable to the situation it refers to, as it may not adequately capture the meaning of something. Therefore, the fuzzy concept can provide greater credibility when an imprecise concept is available, or an ambiguous and imprecise concept is not applicable or insufficient to the situation it refers to. This will be better than not reflect the information in the model. For the sake of simplicity, variables and parameters of the systems are defined exactly in the modeling. However, the estimations of the system parameters and variables may be uncertain or vague in nature since they are found by some experiment, observation, or experience. Hence, when some vague and imprecise information about the parameters is given, then some or all the parameters can be represented by fuzzy numbers, which was first introduced by Zadeh (1996) in 1965, to overcome these uncertainties. Fuzziness allows for the inclusion of vague human assessments in problems. Moreover, it provides an effective way for better assessment of options. Therefore, fuzziness is applicable for many people involved in research and development including engineers, mathematicians, natural scientists, medical researchers, computer software developers, social scientists, business analysts, and jurists.

Fuzzy numbers can be used in place of crisp numbers in the cases of the imprecision that may follow from the lack of exact information, changeable economic conditions, etc. Thus, a crisp SLE becomes a Fuzzy SLE (FSLE) or a Fully FSLE (FFSLE). The difference between the FSLE and the FFSLE is that the coefficient matrix or the variables are considered as crisp in the fuzzy system; but in the fully fuzzy system, all parameters and variables are in the form of fuzzy numbers. Solution methods for the general FSLE have been developed either from modification of the methods proposed for solving the crisp systems or from the development of new solution techniques. In this context of the FSLE, studies using direct and iterative methods can be accessed in the review (Kocken & Albayrak, 2015).

When modeling the engineering problems, it often happens that an SLE also occurs on C, as in vector spaces on R; that is, a system whose both coefficients and variables are complex numbers. Accordingly, Complex SLE (CSLE) are important large-scale applied problems in modeling such as optimization, flow, economics, computational electrodynamics, quantum mechanics, electromagnetism, structural dynamics, electric power system models, wave propagation, magnetized multicomponent transport. A general CSLE can be expressed as $CZ = W$, where C and W are crisp complex matrices and Z is the unknown complex vector, and the system can also be solved via direct or iterative methods, and these systems can be solved via direct or iterative methods such as for crisp real systems.

For the sake of convenience, in some scientific areas such as wave function in quantum mechanics, circuit analysis, etc. some parameters or variables are defined as complex numbers. However, these parameters or variables may take on uncertain values in actual practice. To overcome the uncertainty is appropriate to use fuzzy complex numbers instead of complex numbers in such models. Thus, it is important to develop mathematical models that would appropriately treat fuzzy complex linear systems. For instance, circuits can be modeled in the form of the Fuzzy CSLE (FCSLE). Uncertainty in circuit parameters and environmental conditions leads to the development of a new method that considers

Related Content