Portfolio Optimization for the Indian Stock Market

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INTRODUCTION

The design of optimized portfolios has remained a research topic of broad and intense interest among the researchers of quantitative and statistical finance for a long time. An optimum portfolio allocates the weights to a set of capital assets in a way that optimizes the return and risk of those assets. Markowitz in his seminal work proposed the mean-variance optimization approach which is based on the mean and covariance matrix of returns (Markowitz, 1952). The algorithm, known as the *critical line algorithm* (CLA), despite the elegance in its theoretical framework, has some major limitations. One of the major problems is the adverse effects of the estimation errors in its expected returns and covariance matrix on the performance of the portfolio.

The *hierarchical risk parity* (HRP) and *hierarchical equal risk contribution* (HERC) portfolios are two well-known approaches of portfolio design that attempt to address three major shortcomings of quadratic optimization methods which are particularly relevant to the CLA (de Prado, 2016). These problems are, instability, concentration, and under-performance. Unlike the CLA, the HRP algorithm does not require the covariance matrix of return values to be invertible. The HRP is capable of delivering good results even if the covariance matrix is ill-degenerated or singular, which is an impossibility for a quadratic optimizer. On the other hand, the HERC portfolio optimization adapts the HRP approach to achieve an equal contribution to risk by the constituent stocks in a cluster after forming an optimal number of clusters among a given set of capital assets. Interestingly, even though CLA's objective is to minimize the variance, portfolios formed based on HRP and HERC methods are proven to have a higher likelihood of yielding lower out-of-sample variance than the CLA. The major weakness of the CLA algorithm is that a small deviation in the forecasted future returns can make the CLA deliver widely divergent portfolios. Given the fact that future returns cannot be forecasted with sufficient accuracy, some researchers have proposed risk-based asset allocation using the covariance matrix of the returns. However, this approach brings in another problem of instability. The instability arises because the quadratic programming methods require the inversion of a covariance matrix whose all eigenvalues must be positive. This inversion is prone to large errors when the covariance matrix is numerically ill-conditioned, i.e., when it has a high condition number (Baily & de Prado, 2012). The HRP and HERC portfolios are two among the new portfolio approaches that address the pitfalls of the CLA using techniques of machine learning and graph theory (de Prado, 2016). While HRP exploits the features of the covariance matrix without the requirement of its invertibility or positive-definiteness and works effectively on even a singular covariance matrix of returns, the HERC portfolio leverages the formation of an optimal number of clusters

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among a set of capital assets in a manner that ensures equal risk contribution by the assets in the same cluster (Raffinot, 2018).

Despite being recognized as two approaches that outperform the CLA algorithm, to the best of our knowledge, no study has been carried out so far to compare the performances of the HRP and the HERC portfolios on Indian stocks. This chapter presents a comparative analysis of the performances of the HRP and the HERC portfolios on some important stocks from selected eight sectors listed in the National Stock Exchange (NSE) of India. Based on the report of the NSE on Oct 29, 2021, the most significant stocks of seven sectors and the 50 stocks included in the NIFTY 50 are first identified (NSE, 2021). Portfolios are built using the HRP and the HERC approaches for the eight sectors using the historical prices of the stocks from Jan 1, 2016, to Dec 31, 2020. The portfolios are backtested on the in-sample data of the stock prices from Jan 1, 2016, to Dec 31, 2020, and also on the out-of-sample data of stock prices from Jan 1, 2021, to Nov 1, 2021. Extensive results on the performance of the backtesting of the portfolios are analyzed to identify the better-performing algorithm for portfolio design.

RELATED WORK

Several approaches have been proposed by researchers for accurate prediction of future values of stock prices and using the forecasted results in building robust and optimized portfolios that optimize the returns while minimizing the associated risk. Time series decomposition and econometric approaches like ARIMA, Granger causality, VAR are some of the most popular approaches to future stock price predictions which are used for robust portfolio design (Sen, 2018b, 2017a, 2017b; Sen & Datta Chaudhuri, 2018, 2017b, 2017c, 2016a, 2016b, 2016c). The use of machine learning, deep learning, and reinforcement learning models for future stock price prediction has been the most popular approach of late (Bao et al., 2017; Binkowski et al., 2018; Mehtab & Sen, 2022, 2020a, 2020b, 2020c, 2020d; Mehtab et al., 2021, 2020e; Sen, 2018a; Sen & Datta Chaudhuri, 2017a; Sen & Mehtab, 2021b, 2021c; Sen et al., 2020, 2021e). Hybrid models are also proposed that utilize the algorithms and architectures of machine learning and deep learning and exploit the sentiments in the textual sources on the social web (Audrino et al., 2020; Bollen et al., 2011; Carta et al., 2021; Chen et al., 2019; Galvez & Gravano, 2017; Mehtab & Sen, 2019; Weng et al., 2017). The use of metaheuristics algorithms in solving multi-objective optimization problems for portfolio management has been proposed in several works (Chen et al., 2018; Corazza et al 2021; Macedo et al., 2017; Zhao et al 2020). Several modifications of Markowitz's minimum variance portfolio theory have been advocated by imposing a constraint of the purchase limits and on the cardinality (Clarke et al., 2011; DeMiguel et al., 2009; Qu et al., 2017; Reveiz-Herault, 2016; Saborido et al., 2016; Silva et al., 2015; Syrovatkin, 2020; Vercher and Bermudez, 2015; Zhang et al., 2019). The use of *fuzzy logic*, *genetic algorithms* (GAs), algorithms of *swarm intelligence* (SI), e.g., *particle swarm optimization* (PSO), are also quite common in portfolio optimization (Garcia et al., 2018; Ertenlice & Kalayci, 2018; Erwin & Engelbrecht, 2020). The performances of the mean-variance, eigen, and HRP portfolios have been compared on different stocks from various sectors of the Indian stock market (Sen & Dutta, 2022; Sen & Mehtab, 2021a; Sen et al., 2021d, 2021f, 2021h, 2021i, 2021j). The use of *generalized autoregressive conditional heteroscedasticity* (GARCH) in estimating the future volatility of stocks and portfolios has also been illustrated (Sen et al., 2021g).

The current work presents two methods, the HRP and the HERC portfolio design approaches, to introduce robustness while maximizing the portfolio returns for eight sectors of the NSE of India. Based on the past prices of the stocks from Jan 2016 to Dec 2020, portfolios are designed using the HRP and