Chapter 3
Basic Compartmental Models

ABSTRACT

In this chapter, the compartmental models for disease transmission are described and analyzed. At the outset, models for epidemics are illustrated, showing how to calculate the basic reproduction number and the final size of the epidemic. The authors also study models with multiple compartments, including treatment or isolation of infectives. Subsequently, they consider models including births and deaths in which there may be an endemic equilibrium and study the asymptotic stability of equilibria. They conclude by studying the age of infection models which give a unifying framework for more complicated compartmental models.

INTRODUCTION

The diseases such as influenza, tuberculosis, and measles which belong to the category of communicable diseases have become a part of human life. For most such diseases, the factors that contribute to the spread and transmission are now known. Communicable diseases like measles, influenza, and tuberculosis have become a part of human life. The cause and mechanism of transmission and spread of infections are now known for most diseases. In general, diseases such as measles, chicken pox, influenza, and rubella (German measles) that are transmitted by viral agents, confer immunity against reinfection, whereas diseases such as meningitis, tuberculosis, and gonorrhea have no immunity and recur again unless preventive measures are taken against. There are

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diseases like Malaria and Syphilis that are transmitted through agents (known as vectors), namely mosquitoes and humans respectively.

Epidemic diseases are of more concern to us because they inflict heavy casualties on humanity.

In 1918, the outbreak of Spanish flu caused a havoc with millions of deaths. An endemic disease is one that is always present in a community or in a region. Endemic diseases occur more in underdeveloped countries due to poor hygienic conditions and a lack of advanced health care facilities. Millions of people die of measles, respiratory infections, diarrhea, and other diseases that can be treated or prevented easily. On the contrary, in the western world, these diseases are more uncommon and are not considered as dangerous at all. The high mortality rates due to disease have a significant impact on the economy in afflicted countries.

So in this chapter, our aim is to furnish an introduction to mathematical epidemiology, incorporating the development of mathematical models for the transmission and spread of the disease and the tools required for the analysis of the models. To obtain information about the spread and mechanism of a disease, scientific experiments are performed setting and testing hypotheses. Very often, it is difficult or nearly impossible to design scientific experiments with desired freedom and control due to some serious ethical questions. Sometimes, the data obtained from the reports of the statistics of the disease falls short of the desired accuracy or is incomplete. Besides, the presence of some irregularities such as chaotic behavior in the data leads to misinterpretation of the data, complicating the estimation of parameters and model fitting. This gives rise to the question of whether the mathematical modeling of epidemiology is reliable enough to work on.

The mathematical modeling provides insight into the behavior of the disease, that is, it sheds light on the underlying mechanisms that activate the spread of the disease, and in the process, it gives clues for control strategies. The errors that creep in due to lack of clarity in the experimental data-often the data is nonreproducible and the insufficiency of data points, are often identified in the mathematical modeling. For instance, in most mathematical epidemiologic models whether it be homogeneous or heterogenous, the fundamental results usually show the threshold behavior. That is, if the number of secondary infections caused by a single infective introduced into a wholly susceptible population, called the reproduction number, is less than 1, the disease dies out whereas if it is greater than 1 there will be an increase