Chapter 11
The Evolution of Image Denoising From Model-Driven to Machine Learning: A Mathematical Perspective

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ABSTRACT

Image denoising is a class of image processing algorithms that aim to enhance the visual quality of the acquired images by removing noise inherent in them and is an active area of research under image enhancement and reconstruction techniques. Traditional model-driven methods are motivated by statistical assumptions on data corruption and prior knowledge of the data to recover while the machine learning (ML) approaches require a massive amount of training data. However, the manual tuning of hyperparameters in model-driven approaches and susceptibility to overfitting under learning-based techniques are their major flaws. Recent years have witnessed the amalgamation of both model and ML-based approaches. Infusing model-driven Bayesian estimator in an ML-based approach, supported by robust mathematical arguments, has been shown to achieve optimal denoising solutions in real time with less effect of over-fitting. In this chapter, the evolution of image denoising techniques is covered from a mathematical perspective along with detailed experimental analysis for each class of approach.

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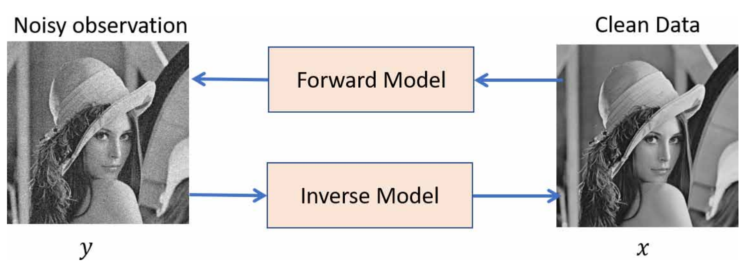
INTRODUCTION: IMAGE DENOISING AS AN INVERSE PROBLEM

Given the parameters *m*, the computation of data (observations) is given by forward model in Figure 1 (left). On the other hand, in order to compute the parameters from data (observations), it is given by inverting the forward model. Here, a model acts as a bridge between data and parameters (see Figure 1 (right)). In other words, finding the effect from the cause is the forward model and finding the causal factors given the effect is the inverse modelling or simply an inverse problem. From the imaging restoration point of view, given a noisy image *y*, estimating the clean image *x* falls under the domain of inverse transformation; as shown in Figure 2.

Figure 1. Forward and inverse model

Figure 2. Forward and inverse modelling in image denoising problem.



Mathematically speaking, considering an image corrupted by forward observation operator H and noise term η, image degradation model is given by:

$$y = Hx \odot \eta \quad (1)$$

However, under the denoising scenario (as the scope of this chapter is noise removal), the term $H=I$ where I is the identity matrix having the same dimension as H. In other words, Equation (1) can be seen as a generalization of different inverse problems under image restoration where H can take different forms depending on whether the given problem is a denoising, deconvolution, super-resolution or image inpainting problem. $\odot$ is an operator whose exact form depends on whether the noise term η is additive or multiplicative. More about it is discussed in the section: N*oise modelling and probabilistic assumptions.*

There is another way in which an inverse problem can be defined, using a terminology known as *well-posed problem*. According to the definition provided by Jacques Hadamard, a problem is said to be well-posed if: