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A New Approach Towards Intuitionistic Fuzzy Multisets

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ABSTRACT

In this paper, a new definition of intuitionistic fuzzy multisets (IFMS) has been introduced. Algebraic operations on these intuitionistic fuzzy multisets are defined, and their properties under these algebraic operations are studied. The author has also introduced a new notion of complement for an IFMS in which the complement of the original set is also an IFMS. The notion of distance and similarity between two IFMSs has been defined, and their properties have also been studied here. An application of IFMS in solving a medical diagnosis problem has been provided at the end.

KEYWORDS

Cardinality, Complement, COVID-19, Diagnosis, Distance, Fuzzy Multisets, Intuitionistic Fuzzy Multisets, Multisets, Similarity

1. INTRODUCTION

Multisets are useful generalizations of classical sets that have a variety of applications in the field of computer science such as information retrieval on internet etc. Kunth (1969) introduced the notion of multisets where one element of the universe may repeat itself several times in a multiset and a 'count function' notes the number of such repetitions. Actually this count function in a multiset is the generalization of the characteristic function for classical sets. After Kunth several authors studied the theory and application of multisets (Blizard, 1989; Hickman, 1980; Nazmul & Majumdar, 2013). Fuzzy multisets (Yagar, 1986; Blizard, 1989; Miyamoto, 2001; Syropoulos, 2012) are again generalizations of crisp multisets in light of fuzzy sets (Zadeh, 1965) where we have a decreasing sequence of membership degrees for each element in a multiset. Further generalizations of fuzzy multisets which is based on Atanasov's intuitionistic fuzzy sets (1986) are called intuitionistic fuzzy multisets. IFMS's were first introduced by Shinoj and John (2012, 2013, 2015). In intuitionistic fuzzy multisets, we have two sequences of numbers, namely a membership sequence which is decreasing in nature and a non-membership sequence against each element of the universe that is present in the multiset. The non-membership degree sequence is not ordered and the sum of the corresponding elements of these two sequences satisfies the intuitionistic condition, i.e. the sum of these two degrees must be less or equal to 1. Several authors (Ejegwa 2016) have further studied the theory and applications of intuitionistic fuzzy multisets in recent times. But the main problem with fuzzy multisets and intuitionistic fuzzy multisets are their complement. In both cases the membership sequence is decreasing in order, therefore the natural fuzzy or intuitionistic fuzzy complement will not preserve the order of this sequence. That is why we will not get a fuzzy or intuitionistic fuzzy multiset when we take the natural complement. This paper aims to address this issue in case of intuitionistic fuzzy multisets. Again the notion of distance and similarity between any two sets are a very useful concept

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in mathematics and they have a number of important applications. Smidtz & Krapyzik (2000) first introduced the notion of distance between two intuitionistic fuzzy sets. Here we have introduced the notion of distance and similarity between two IFMSs and also shown a possible application of IFMS in solving a medical diagnosis problem. The rest of this paper is constituted as follows: In section 2, we give some preliminary definitions and properties related to intuitionistic fuzzy multisets that are required to understand this paper. A new notion of IFMSs has been defined in section 3 and some basic algebraic operations have been defined on them. Section 4 introduces the notion of distance between two IFMSs and studies its properties. Also a distance based measure of similarity between two IFMSs has been defined here. An application of IFMS in medical diagnosis problem has been discussed in section 5. Section 6 concludes the paper.

2. PRELIMINARIES

In this section some preliminary definitions and results regarding multisets and its generalizations are given.

Definition 2.1 (Multisets) (Miyamoto, 2001) A multiset M of the universe X is characterized by the count function C_M: X→N, where N= {0, 1, 2, ...}. Thus, $C_M(x)$ is the number of occurrences of the element x of the universe X.

Definition 2.2 (Fuzzy Multisets) (Yagar, 1986) A fuzzy multiset (FMS) A drawn from the universe X is characterized by a function 'count membership' of A denoted by CM_A such that CM_A:X→P([0,1]). Thus for each element x of the universe X, we get a membership sequence which is defined as a decreasingly ordered sequence of elements from [0, 1]. Thus

$CM_A(x) = (\mu_A^1, \mu_A^2, ..., \mu_A^p)$, *where* $\mu_A^1 \geq \mu_A^2 \geq ..... \geq \mu_A^p$, *where* p is a finite positive integer.

Definition 2.3 (Intuitionistic fuzzy sets) (Atanassov, 1986) An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) >; x \in X\}$ where the functions $\mu_A(x), \nu_A(x) : X \to [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$.

Definition 2.4 (Intuitionistic fuzzy multisets) (Shinoj & John, 2013) An IFMS A over X is characterized by two functions: 'count membership' of A (denoted by CM_A) and 'count non-membership' of A (denoted by CN_A) given respectively as: CM_A, CN_A: X→P([0,1]). Now for each

$x \in X, CM_A(x) = (\mu_A^1, \mu_A^2, ..., \mu_A^p)$, *where* $\mu_A^i \geq \mu_A^i \geq ..... \geq \mu_A^i$
and $CN_A(x) = (\nu_A^1, \nu_A^2, ..., \nu_A^p)$, *with* $0 \leq \mu_A^i + \nu_A^i \leq 1 \; \forall i = 1,2,..., p.$

An IFMS is denoted by $A = \{< x, (\mu_A^1, \mu_A^2, ..., \mu_A^p), (\nu_A^1, \nu_A^2, ..., \nu_A^p) >; x \in X\}$.

Definition 2.5 (Shinoj & John, 2012) The length of an element x in a IFMS A, denoted by L(x, A), is defined as the cardinality of $CM_A(x)$ or $CN_A(x)$, i.e. $L(x,A)=|CM_A(x)|=|CN_A(x)|$.

3. INTUITIONISTIC FUZZY MULTISETS

In this section the author introduces a new definition of IFMS which is different from earlier definitions. He has also defined some basic algebraic operations on them including the notion of complement and properties of IFMS under these operations have also been studied here. Throughout this paper it has been assumed that the universe of discourse is finite.