Chapter I
Foundations of Rough Sets from Vagueness Perspective

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ABSTRACT

We present three types of knowledge that can be specified according to the rough set theory. Then, we present three corresponding types of algebraic structures appearing in the rough set theory. This leads to three following types of vagueness: crispness, classical vagueness, and a new concept of “intermediate” vagueness. We also propose two classifications of information systems and approximation spaces. Based on them, we differentiate between information and knowledge.

INTRODUCTION

Handling vagueness was one of the motivations for proposing the rough set theory (Pawlak, 2004, see also Pawlak, 1982, 1991, 2003). In this chapter, we present algebraic foundations of the rough set theory from that perspective. Vagueness is understood according to the tertium non datur principle from traditional logic; the contemporary version of this principle is called the law of excluded middle (Frege, 1903; Hempel, 1939; Pawlak, 2004; Russell, 1923). Concepts in the rough set theory are represented by subsets of a universe of discourse. Each concept, $X$, can be represented by two subsets: that consisting of examples of $X$ (i.e., objects, which can be certainly classified as belonging to $X$) called a positive region of $X$, and that consisting of objects that certainly do not belong to $X$, called a negative region of $X$. Positive region of a given concept is represented...
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by its lower approximation. Negative region is represented as a complementation of an upper approximation of the set. A complementation of the union of these two sets is called a boundary. Thus, boundary consists of objects that cannot be certainly classified as belonging to a concept, or as not belonging. A concept is vague if its boundary is nonempty, otherwise it is crisp (Pawlak, 2004). Vague concepts are represented in the rough set theory as rough sets. Crisp concepts satisfy the tertium non datur principle, their boundaries are empty, there are no objects of third type or, in other words, the middles between their positive and negative regions are excluded. Rough sets (representing vague concepts) violate this principle. However, classical rough sets, based on equivalence relations, satisfy the weak law of excluded middle. This law states that a pseudocomplement of a set $X$ with a pseudocomplement of a pseudocomplement of a set $X$ is equal to whole space or, in more general version it states that a pseudocomplement of an element $x$ of a pseudocomplemented lattice $L$ with a pseudocomplement of a pseudocomplement of this element $x$ is equal to unit of the lattice $L$. This observation gives a reason for differentiating two types of vagueness: strong vagueness (or simply vagueness) and weak vagueness. Weakly vague concepts satisfy the weak law of excluded middle, while strongly vague concepts violate this law. Concepts of both types violate the law of excluded middle.

We show that an incompleteness of information is a source of these two types of vagueness.

Incomplete information, represented by deterministic incomplete information systems, can be analyzed by means of classical approximation spaces. Such spaces consist of a universe and an indiscernibility relation, which is always equivalence relation. Knowledge granules determined by these spaces are equivalence classes. Rough sets within classical approximation spaces violate the law of excluded middle, but they satisfy the weak law of excluded middle. Thus, such rough sets represent vague concepts in an “intermediate” sense.

Nondeterministic information systems gave a reason for introducing many information relations (Orłowska, 1993, 1997, 1998). Most of these relations are tolerance relations, which are not necessarily transitive, that is are not equivalence relations. Therefore, incomplete information, represented by nondeterministic information systems, can be analyzed by tolerance spaces that consist of a universe and a tolerance relation. Tolerance spaces admit various ways of granulations of the universe. We present a type of knowledge granulations of the tolerance spaces such that if a tolerance relation is not transitive, then rough sets determined by this granulation violate even the weak law of excluded middle. Thus, such rough sets represent vague concepts in the strong sense.

We propose a differentiation between information and knowledge. It follows from foundations of the rough set theory. Starting point is reflected by two suggested classifications of information systems and approximation spaces. We point out that properties of incompleteness and determinacy of information systems are independent and, particularly, that there are complete indeterministic information systems. Information, represented in both complete deterministic and complete indeterministic information systems, enables us to discern between two arbitrary objects from the universe. So, indiscernibility relations determined by information systems of these types are identity relations. Since, according to Pawlak (1982, 1991, 2004), knowledge is based on ability to discern between objects, then complete information systems are bases for constructing complete knowledge, which is represented by identity relations. So, knowledge has more general nature than information. Complete knowledge is necessarily exact; thus, there is no complete inexact knowledge. It follows that properties of completeness and exactness of knowledge are
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