Chapter III
Filters in Multirate Systems

INTRODUCTION

The role of filtering in sampling-rate conversion has been considered in Chapter II. The importance of filtering arises from the fact that the sampling theorem should be respected for all the sampling rates of the system at hand. Filters are required to bandlimit the spectrum of the signal to the prescribed bandwidth in accordance with the actual sampling rate. In sampling rate conversion systems, filters are used in decimation to suppress aliasing and in interpolation to remove imaging. Since an ideal frequency response cannot be achieved, the performance of the system for sampling rate conversion is mainly determined by filter characteristics. Obviously, an appropriate filter should enable the sampling rate conversion with minimal signal distortion. The main advantage of a multirate system is the computational efficiency, and therefore, a decimator (interpolator) that implements a high-order digital filter could not be tolerated. The specific role of a digital filter in sampling rate conversion demands high-performance filtering with the lowest possible complexity. To reach this goal one has to concentrate first on the choice of the appropriate design specifications in order to provide minimal signal distortion. Secondly, the multirate filter is to be designed in a manner to satisfy the prescribed characteristics and to provide a low-complexity implementation structure.

In this chapter, we discuss first the spectral characteristics of decimators and interpolators and introduce three commonly used types of filter specifications. In the sequel, we review the MATLAB functions that are appropriate for the design of FIR and IIR filters to satisfy the specifications. An approach to computation of aliasing characteristics of decimators is given and illustrated by examples. This chapter considers also the analysis of sampling rate conversion for band-pass signals. Chapter concludes with MATLAB exercises for individual study.
SPECTRAL CHARACTERISTICS OF DECIMATORS AND INTERPOLATORS

The sampling rate reduction and sampling rate increase have been discussed in Chapter II. When reducing sampling rate, filtering should precede the sampling rate reduction to suppress aliasing. This filter, called an antialiasing filter, attenuates the frequency components outside the new baseband of the signal that is, bandlimits the signal spectrum to a half of the new sampling frequency. When increasing sampling rate, filtering follows the up-sampling operation. The role of the filter is to attenuate unwanted periodic spectra which appear in the new baseband. These periodic spectra are called ‘images’, and the filter, which is used to remove them, is called an antiimaging filter.

As introduced in Chapter II, the sampling rate reduction is called decimation, and the sampling rate increase is called interpolation. The performance of a decimator or an interpolator is mainly determined by filter characteristics. Therefore, the specifications for filter design should be a reasonable compromise between the performance of a decimator (interpolator) and the filter complexity.

Let us consider in more detail the unwanted spectra produced by a down-sampling operation that should be attenuated by an antialiasing filter. The structure of a factor-of-M decimator consisting of the antialiasing filter $H(z)$ and a factor-of-M down-sampler is sketched in Figure 3.1. The input signal $\{x[n]\}$ operating at the sampling frequency $F_x$ is filtered by the antialiasing filter $H(z)$ giving the signal $\{v[n]\}$. The output signal $\{y[n]\}$ is obtained by picking up every $M^{th}$ sample of $\{v[n]\}$. The sampling frequency is reduced by $M$, $F_y = F_x / M$.

The role of the antialiasing filter $H(z)$ is to bandlimit the spectrum of the input signal to a half of the new sampling rate. The frequency response of an ideal linear-phase antialiasing filter $H_0(e^{j\omega})$ is defined as,

$$H_0(e^{j\omega}) = \begin{cases} e^{-jK\omega}, & 0 \leq \omega \leq \omega_p, \\ 0, & \omega_p \leq \omega \leq \pi. \end{cases} \quad (3.1)$$

where $\omega_p$ is the cutoff, and $K$ is a positive constant.

If the phase characteristic is not of importance, the ideal characteristic is defined only in terms of the magnitude response,

$$|H_0(\omega)| = \begin{cases} 1, & 0 \leq \omega \leq \omega_p, \\ 0, & \omega_p \leq \omega \leq \pi. \end{cases} \quad (3.2)$$

The ideal characteristics defined above can be appropriately approximated with real filters. For determining the specifications for $H(z)$, we need to identify the frequency bands in which the spectrum of the input signal should be attenuated. As shown in Chapter II, the spectrum of the down-sampled signal

Figure 3.1. Structure of a factor-of-M decimator
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