Chapter II
Basics of Multirate Systems

INTRODUCTION

Linear time-invariant systems operate at a single sampling rate i.e. the sampling rate is the same at the input and at the output of the system, and at all the nodes inside the system. Thus, in an LTI system, the sampling rate doesn’t change in different stages of the system. Systems that use different sampling rates at different stages are called the multirate systems. The multirate techniques are used to convert the given sampling rate to the desired sampling rate, and to provide different sampling rates through the system without destroying the signal components of interest.

In this chapter, we consider the sampling rate alterations when changing the sampling rate by an integer factor. We describe the basic sampling rate alteration operations, and the effects of those operations on the spectrum of the signal.

TIME-DOMAIN REPRESENTATION OF DOWN-SAMPLING AND UP-SAMPLING

Converting the sampling rate means that one discrete signal is converted into another discrete signal with a different sampling rate. Two discrete signals with different sampling rates can be used to convey the same information. For example, a bandlimited continuous signal $x(t)$ might be represented by two different discrete signals $\{x[n]\}$ and $\{y[n]\}$ obtained by the uniform sampling of the original signal $x(t)$ with two different sampling frequencies $F_T$ and $F_{T'}$.

\[
x[n] = x_c(nT) \quad \text{and} \quad y[n] = x_c\left(nT'\right)
\]

(2.1)

where $T = 1/F_T$ and $T' = 1/F_{T'}$ are the corresponding sampling intervals. When the sampling frequencies $F_T$ and $F_{T'}$ are chosen in such a way that each of them exceeds at least two times the highest frequency
in the spectrum of $x(t)$, the original signal $x(t)$ can be reconstructed from either $\{x[n]\}$ or $\{y[n]\}$. Hence, the two signals operating at two different sampling rates are carrying the same information. By using the discrete-time operations, signal $\{x[n]\}$ can be converted to $\{y[n]\}$, or vice versa, with minimal signal distortions.

The basic operations in sampling rate alteration process are the sampling rate decrease and the sampling rate increase. Employing two operators can perform the sampling rate alteration: a down-sampler for the sampling rate decrease, and an up-sampler for the sampling rate increase. The down-sampler and the up-sampler are the sampling rate alteration devices since they decrease or increase the sampling rate of the input sequence.

**Down-Sampling Operation**

The *down-sampling* operation with a down-sampling factor $M$, where $M$ is a positive integer, is implemented by discharging $M-1$ consecutive samples and retaining every $M$th sample. Applying the down-sampling operation to the discrete signal $\{x[n]\}$, produces the down-sampled signal $\{y[m]\}$

$$\{y[m]\} = \{x[mM]\}. \quad (2.2)$$

The down-sampling can be imagined as a two-step operation. In the first step, the original signal $\{x[n]\}$ is multiplied with the sampling function $\{s_M[n]\}$ defined by,

$$s_M[n] = \begin{cases} 1, & n = 0, \pm M, \pm 2M, \ldots, \\ 0, & \text{otherwise} \end{cases}. \quad (2.3)$$

Multiplying the sequence $\{x[n]\}$ by the sampling function $\{s_M[n]\}$ results in the intermediate signal $\{y_s[m]\}$,

$$y_s[n] = x[n]s_M[n] = \begin{cases} x[n], & n = 0, \pm M, \pm 2M, \ldots, \\ 0, & \text{otherwise} \end{cases}. \quad (2.4)$$

This operation is called a *discrete sampling*. In the second step, the zero valued samples in $\{y_s[m]\}$ are omitted resulting in the down-sampled sequence $\{y[m]\}$,

$$y[m] = y_s[mM] = x[mM]. \quad (2.5)$$

Figure 2.1 illustrates the two-step description of the down-sampling operation explained above for the example down-sampling factor $M = 3$.

The down-sampling operation is sometimes called the *signal compression*, and the down-sampler is also known as a *compressor*. A block diagram representing the down-sampling operation is shown in Figure 2.2. The box with a down pointed arrow followed with the factor $M$ is used to symbolize the down-sampling operation.

Figure 2.3 illustrates the time dimensions of down-sampling. This operation reduces the sampling frequency $F_T$ of the original signal $\{x(nT)\}$. The sampling frequency $F_T'$ of the signal $\{y(mT')\}$ is $M$ times smaller than the sampling frequency of the original signal, i.e, $F_T' = F_T/M$.  

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