Chapter 15
Robust Control Methods for Finite Time Synchronization of Uncertain Nonlinear Systems

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ABSTRACT

This chapter addresses the dynamic analysis and two different control strategies for the synchronization of new topology of Colpitts oscillator submitted to uncertainties and external disturbances. The diagrams obtained reveal precisely spirals bifurcation and chaos when for a specific values of the system parameters. Based on the relevant control, the authors have controlled this striking phenomenon in the system. The first (control) deals with the sliding mode control (SMC) method. Some important aspects of the design and implementation are considered to reach a suitable controller for the applications. The second presents an adaptive robust tracking control strategy based on a modified polynomial observer which tends to follow exponentially the chaotic Colpitts circuits brought back to a topology of the Chua oscillator with perturbations. To highlight the contribution, they also present some simulation results with the purpose to compare the proposed method to the classical polynomial observer.

1. INTRODUCTION

The study of nonlinear systems is very relevant because the nature and all its phenomena are intrinsically nonlinear. The understanding of nonlinear systems is a continuous challenge and requires an interdisciplinary approach that brings together methodologies and tools from different research areas, and also inspires the development of new approaches based on the theory of dynamical systems. Many nonlinear phenomena are described by differential equations, especially by partial differential equations (PDEs) and ordinary differential equations (ODEs). Note that, symmetry group analysis of a differential equa-

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tion appears as a powerful method to analyze PDEs (Ibragimov, 1985; Olver, 1993). Among its many applications, we highlight the fact that they allow us to obtain exact solutions of a PDE, directly or by using similarity solutions (Avdonina et al., 2013); classify invariant equations; reduce the number of independent variables and construct conservation laws. Many approaches to analyze the ODEs are developed in literature (Kudryashov et al., 2005, Kudryashov, 2013). Due to implementation of sophisticated algorithms, the approach is basically qualitative through computer-based methods. Many electronic circuits such as Colpitts oscillators, van der Pol oscillators, Chen systems, Chua circuit, Harley oscillator, Jerk oscillator and so on depicts nonlinear phenomena. Amongst theses topologies, the dynamic of Chua's circuit is better understood due to its piecewise nonlinear continuous function. The impact of nonlinearity on circuit analysis for the design of new electronic devices have generated much interest in nonlinear circuit theory (Vaidyanathan et al., 2017a, 2017b; Volos et al., 2016; Azar et al., 2017a; Pham et al., 2017; Vaidyanathan and Azar, 2016a, 2016b)

The various models associated with the Chua's circuit have been recently found helpful in demonstrating and explaining the various facets of chaos (Chua and Lin, 1900). Attempts are made in the literature to discover new members of the large family of Chua's circuits and to find their close relatives (Chua, 1993). The present communication deals with the well-known Colpitts oscillator, which was shown to be topologically similar to Chua's circuit (Sarafian and Kaplan, 1995). The nonlinearity of the active device in the Colpitts oscillator is modified to be purely odd, then the circuit exhibits chaotic phenomena closely related to those exhibited by the classical Chua's circuit. There exists a robust relationship between the Colpitts oscillator and the Chua's oscillator (Kennedy, 1995), one can show that both systems are not simply close relatives, but they are even strongly related and can be regarded as being conjugate one to another. The dynamics behavior of the Colpitts oscillator that results from this kind of modeling is reported in this paper and is related to the well-known chaotic behavior of Chua's circuit. In the past, these works was essentially theoretical and the technological aspect was limited (Kennedy, 1995). It ought to be mentioned that the relationship between Colpitts and Chua oscillators with a symmetric nonlinearity leads to a singular phenomenon with only very few cases reported. The research on this model is motivated by the discovery of a simple third-order ordinary differential equations of the form $\dddot{x} = J(x, \dot{x}, \ddot{x})$ whose solution are chaotic. The nonlinear function J is called a *Jerk,* because it describes the third-time derivative of acceleration in a mechanical system (Schot, 1978). Sprott (Sprott, 2000) proposed many new jerk systems with several nonlinearities that show chaotic behavior with easy electronic implementation. In one of the chapter of "Elegant Chaos: Algebraically Simple flow", book published in 2010, Sprott proposed a list of 16 autonomous jerk oscillators with different nonlinearities called memory oscillators (MO_0 to MO_{15}) (Sprott, 2010). A simple chaotic jerk circuit was used in a sound encryption scheme (Volos, 2017). Especially, a three-dimensional novel jerk chaotic oscillator with two hyperbolic sinusoidal nonlinearities was reported in (Vaidyanathan, 2014). We should note that, the representation of the Colpitts oscillator as jerky dynamics has another useful and important advantage: based on their jerky dynamics, a classification of different dynamical systems is possible, because the transformation of certain functionally different three-dimensional dynamical systems can lead to the same jerky dynamics (Kammogne et al., 2017; Eichhorn Ralf, et al., 2002).

Synchronization of chaotic systems is a current challenge in control theory (Azar and Vaidyanathan, 2017; Azar and Vaidyanathan, 2015a, 2015b, 2015c; Wang et al., 2017). Researches on the coupled nonlinear oscillators constitute an excellent framework for understanding the various complex collective dynamics that spontaneously emerge in real-life systems (Hossein, et al., 2018; Pikovsky and Rosenblum,