Chapter I
An Introduction to Multi-Objective Optimization

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ABSTRACT

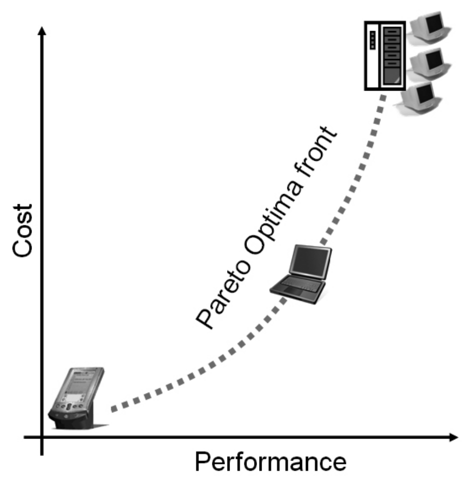
This chapter is devoted to summarize common concepts related to multi-objective optimization (MO). An overview of "traditional" as well as CI-based MO is given. Further, all aspects of performance assessment for MO techniques are discussed. Finally, challenges facing MO techniques are addressed. All of these description and analysis give the readers basic knowledge for understandings the rest of the book.

OVERVIEW

Real-world problems often have multiple conflicting objectives. For example, when purchasing computing equipments, we would usually like to have a high-performance system, but we also want to spend less money buying it (see Figure 1). Obviously, in these problems, there is no single solution that is the best when measured on all objectives (note that the terms *solution, individual* and *point* are used interchangeably in this book). These problems are examples of a special class of optimization problems called multi-objective optimization problems (MOPs). The question is what is an optimal solution for a multi-objective problem? In general, it is called a Pareto optimal solution if there exists no other feasible solution which would decrease some objectives (suppose a minimization problem) without causing a simultaneous increase in at least one other objective (Coello, 2006b).

With this definition of optimality, we usually find several trade-off solutions (called the *Pareto optimal set* to honor Vilfredo Pareto (Pareto,

Figure 1. An example of cost-performance problem



1896), or *Pareto optimal front* (POF) for the plot of the vectors corresponding to these solutions). In that sense, the search for an optimal solution has fundamentally changed from what we see in the case of single-objective problems. The task of solving MOPs is called *multi-objective optimization*.

However, users practically need only one solution from the set of optimal trade-off solutions. Therefore, solving MOPs can be seen as the combination of both searching and decision-making (Horn, 1997). In order to support this, there are four main approaches in the literature (Miettinen, 1999). The first one does not use preference information (called *no-preference*). These methods solve a problem and give a solution directly to the decision maker. The second one is to find all possible solutions of the nondominated set and to then use the user preference to determine the most suitable one (called *decision making after search*, or *posterior*). Meanwhile, the third approach is to incorporate the use of preference before the optimization process; and hence it will result in only one solution at the end (called *decision making before search*, or *priori*). With this approach, the bias (from the user preference) is imposed all the time. The fourth approach (called *decision making during search*, or *interactive*) is to hybridize the second and third ones in which a human decision making is periodically used to refine the obtained trade-off solutions and thus to guide the search. In general, the second one is mostly preferred within the research community since it is less subjective than the other two.

Evolutionary algorithms (EAs) (Back, 1996; Goldberg, 1989; Michalewicz, 1996) have emerged as heuristic and global alternatives with their most striking characteristic being: using a population for the search in each iteration. This makes them suitable for solving multi-objective problems. That is why they have attracted significant attention from the research community over the last two decades. Today, the rise of evolutionary multi-objective optimization can be seen by the number of publications produced over time (Coello, 2006a). It is worthwhile to note that there are several paradigms that have emerged as alternatives for the conventional EAs, such as Particle Swarm Optimization (PSO) (Kennedy & Eberhart, 1995), Ant Colony Optimization (ACO) (Dorigo & Stutzle, 2004), Differential Evolution (DE) (Price, Storn, & Lampinen, 2005), Estimation of Distribution Algorithms (EDA) (Larraanaga & Lozano, 2002), and Artificial Immune Systems (AIS) (Dasgupta, 1998). For them, mutation and crossover operators might be replaced by some specific operator inspired by different phenomena in nature.

This chapter is organized as follows: the second section is for the common concepts and notations in multi-objective optimization using evolutionary algorithms that are used throughout the book. It is followed by descriptions of traditional multi-objective algorithms as well as MOEAs (the third and fourth sections respectively). The fifth and sixth sections are dedicated to the research issues and the performance assessment. The chapter is concluded in the final section.