Chapter 4
Quantum Fourier Transforms

ABSTRACT

Quantum Fourier transform (QFT) plays a key role in many quantum algorithms, but the existing circuits of QFT are incomplete and lacking the proof of correctness. Furthermore, it is difficult to apply QFT to the concrete field of information processing. Thus, this chapter firstly investigates quantum vision representation (QVR) and develops a model of QVR (MQVR). Then, four complete circuits of QFT and inverse QFT (IQFT) are designed. Meanwhile, this chapter proves the correctness of the four complete circuits using formula derivation. Next, 2D QFT and 3D QFT based on QVR are proposed. Experimental results with simulation show the proposed QFTs are valid and useful in processing quantum images and videos. In conclusion, this chapter develops a complete framework of QFT based on QVR and provides a feasible scheme for QFT to be applied in quantum vision information processing.

INTRODUCTION

Quantum Fourier transform (QFT) plays the key role in Shor's prime factorization and discrete logarithms (Shor, 1994). In the aspect of information processing, quantum image algorithms based on QFT are research hotspots, and their examples include the watermarking algorithms of quantum images (Zhang, Gao, Liu, Wen, & Chen, 2013; Zhang, Gao, Liu, Wen, & Chen, 2013), and the encryption and decryption algorithm (Yang, Xia, Jia, & Zhang, 2013; Li, Li, Chen, & Xia, 2018).

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QFT can be classified with 1-dimensional or multi-dimensional according to the types of data it acts on are 1-dimension or multi-dimension. Some quantum circuits of 1D QFT have been designed (Nielsen, & Chuang, 2000; Barenco, Ekert, Suominen, & Törmä, 1996; Karafyllidis, 2003; Wang, Zhu, Zhang, & Yeon, 2011; Heo, Kang, Hong, Yang, & Choi, 2016). In addition, the semi-classical Fourier transform (Coppersmith, 2002) and the approximate Fourier transform (Griffiths, & Niu, 1996) have been proposed. The complexity of 1D QFT implementation on elements is (Nielsen, & Chuang, 2000). In contrast, fast Fourier Transform (Cooley, & Tukey, 1965), one of the best classical algorithms, computes the discrete Fourier transform with the complexity. Thus, QFT achieves exponentially speed up in comparison with its classical counterpart. The classical 2D and 3D discrete Fourier transforms were applied directly to image processing (Park, 2015; He, Zhou, & Cui, 2012), and their quantum counterparts were proposed for quantum image processing (Li, Fan, Xia, Song, & He, 2018).

This chapter describes perfect shuffle permutations, generalized tensor products and their implementation circuits (Li, Fan, Xia, Song, & He, 2018). Next, the implementation circuits of 1D, 2D and 3D QFTs by using perfect shuffle permutations and generalized tensor products (Li, Fan, Xia, Song, & He, 2018).

PERFECT SHUFFLE PERMUTATIONS

The perfect shuffle permutation $P_{n,m}$ is an $mn \times mn$ matrix, which shuffles n packs of m cards into m packs of n cards, and satisfies

$$(P_{n,m})_{k,l} = \delta_{v,z'}\delta_{z,v'}, \tag{5.1}$$

where

$$k = vn + z, \, l = v'm + z',$$

$$0 \le v, z' < m, \, 0 \le v', z < n .$$

$\delta_{x,y}$ is the Kronecker delta function, i.e., $\delta_{x,y}=0$ if $x \neq y$, otherwise $\delta_{x,y}=1$ (Fino & Algazi, 1977).

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