Modeling of Uncertain Nonlinear System With Z-Numbers

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INTRODUCTION

Interpolation technique is extensively implemented in order to function estimation (Boffi & Gastaldi, 2006; Jafarian et al., 2016; Mastylo, 2010). In (Schroeder et al., 1991) a systolic algorithm in order to interpolate as well as evaluate of polynomials by utilizing a linear array of processors is proposed. Two-dimensional polynomial interpolation is described in (Zoli, 2008). In (Neidinger, 2009) a multivariable interpolation method is proposed. In (Olver, 2006), the multivariate Vandermode matrix is implemented. Recently, Smooth function estimation is extensively used (Szabados &Vertesi 1990; Tikhomirov, 1990) which leads a model by using Lagrange interpolating polynomials at the points of product grids (Barthelmann et al., 2000; Xiu & Hesthaven, 2005). However, if there are uncertainties in the interpolation points, none of these techniques will work properly.

The fuzzy equation is taken to be the general form of the fuzzy polynomial. The concept of fuzzy modeling is based on finding the fuzzy coefficients of the fuzzy equation. Various methods have been developed (Goetschel &Voxman, 1986; Kajani et al., 2005; Mamdani, 1976; Mazandarani & Kamyad, 2013; Salahshour et al., 2012; Takagi & Sugeno, 1985; Wang & liu, 2011; Jafari & Yu, 2015a, 2015b, 2015c, 2015d, 2017a; Jafari et al, 2019a, 2016; Razvarz & Jafari, 2017a, 2017b; Razvarz et al., 2017, 2018). In (Friedman et al., 1998) a general fuzzy linear system is studied utilizing the embedding method. In (Buckley & Qu, 1990) the necessary and sufficient conditions for linear as well as quadratic equations are presented in order to contain a solution in a case that the parameters are either real or complex fuzzy numbers. The homotypic analysis technique is discussed in (Abbasbandy, 2006). The Newton's method is proposed in (Abbasbandy & Ezzati, 2006b). In (Allahviranloo et al., 2007), fixed point technique is suggested in order to solve a system of fuzzy nonlinear equations. Nevertheless, these techniques are so much complex.

DOI: 10.4018/978-1-7998-3479-3.ch022

Iterative technique (Llanas & Sainz, 2006), interpolation technique (Waziri & Majid, 2012) and Runge-Kutta technique (Pederson & Sambandham, 2008) can be applied for obtaining the numerical solution of the fuzzy equation. These techniques can also be applied to fuzzy differential equations (Kajani & Lupulescu, 2009). Neural networks can also be applied in order to find the solutions of the fuzzy equations (Cybenko, 1989; Ito, 2001; Jafari & Razvarz, 2017, 2018, 2015e; Jafari & Yu, 2017b, 2017c; Jafari et al., 2018a, 2018b, 2019b; Jafarian et al., 2016; Jafarian & Jafari, 2012; Yu & Jafari, 2019). In (Buckley & Eslami, 1997), neural network technique is utilized in order to solve the fuzzy quadratic equation. In (Jafari & Razvarz, 2017; Jafarian et al., 2012), the neural network is used in order to obtain the numerical solution of the dual fuzzy equation. In (Mosleh, 2013) neural network method is utilized in order to find the approximate solution of the fully fuzzy matrix equation.

In this book chapter, the neural network technique is utilized in order to estimate the Z-number coefficients of fuzzy equations. The concept of Z-number is used in various areas related to the analysis of the decisions (Kang et al., 2012a; Zadeh, 2006). Z-number is more precise when compared with the fuzzy number. Fewer studies are reported concerned to the topic of Z-numbers (Gardashova, 2014). Converting Z-number into the usual fuzzy set is discussed in (Kang et al, 2012b). In (Aliev et al., 2015) the theoretical basics of the arithmetic of discrete Z-numbers are presented.

BACKGROUND

This work is the first attempt in finding the Z-number coefficients of fuzzy equations. In order to train the neural network, the backpropagation approach is used. The work is organized as follows. In Section 2, some basic definitions related to the Z-numbers are given. The proposed method for obtaining the Z-number coefficients of the fuzzy equations is demonstrated in Section 3. Some examples with applications in mechanics is given in Section 4. Section 5 concludes the work and provides discussions on further work.

Nonlinear System Modeling With Fuzzy Equations and Z-Numbers

Provide A common discrete-time nonlinear system is depicted as

$$\vartheta_{r+1} = f\left[\vartheta_r, q_r\right], \quad w_r = g\left[\vartheta_r\right] \tag{1}$$

where $q_r \in \Re^u$ is the input vector, $\vartheta_r \in \Re^l$ is an internal state vector, also $w_r \in \Re^m$ is the output vector. f as well as g are generalized nonlinear smooth functions f,g∈C^∞. Define $W_r = \left[w_{r+1}^T, w_r^T, \cdots\right]^T$ as well as $Q_r = \left[q_{r+1}^T, q_r^T, \cdots\right]^T$. Assume $\frac{\partial W}{\partial \vartheta}$ is non-singular at the instance $\vartheta_r = 0$, Q_r=0, so the following model is extracted

$$w_r = \Upsilon[w_{r-1}^T, w_{r-2}^T, \cdots q_r^T, q_{r-1}^T, \cdots] \tag{2}$$

in which $\Upsilon(\bullet)$is a nonlinear difference equation representing the plant dynamics, q_r as well as w_r are calculable scalar input and output respectively.