

# Chapter 3

## Visualisation of Mathematical Thinking

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### ABSTRACT

*Drawing is not proving. For a long time, this argument has been used to avoid the use of visualisation in mathematics. Nevertheless, a number of proofs, concepts, and ideas are easier to understand with the help of a small drawing. In this chapter, the authors show that visualisation in mathematics is helpful not only to illustrate but also to create ideas, and this at all levels.*

### INTRODUCTION

The scene took place at the time of my studies. My professor was at the blackboard, in a packed lecture hall. In front of his fascinated students, he was proving a deep theorem of geometry, using number of diagrams he drew with confidence. The blackboard was becoming white but, suddenly, he stopped in the middle of a diagram (see Figure1).

*Figure 1. Hervé Lehning, New Math Diagram*  
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$$\begin{array}{ccccc}
 H(X, A) & \longrightarrow & H(X, V) & \longleftarrow & H(X-A, V-A) \\
 \downarrow q & & \downarrow q & & \downarrow q \\
 H(X/A, V/A) & \longrightarrow & H(X, & & 
 \end{array}$$

As time went by, the professor looked more and more puzzled. After a while, he started to make a little drawing, unfortunately hidden by his body. Suddenly, he looked illuminated, erased his drawing and resumed his proof with number of diagrams; we noted them without understanding well. At the end

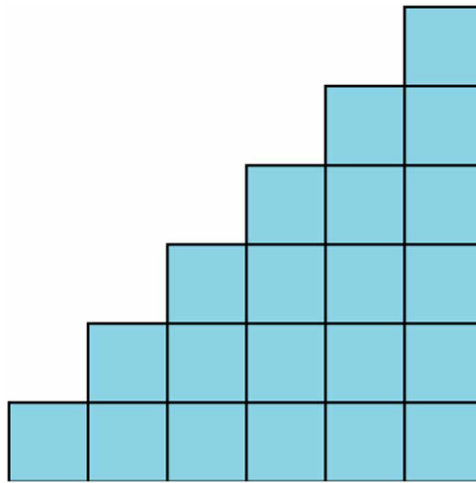
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of the lecture, the bravest students went to the desk to ask him some explanations about his little drawing. His reply was unequivocal: “there’s no question to fill your spirit with bad habits of thought”. His reason to refuse was his pedagogical ideas: we must be freed of the errors of the past, and among them of the habit of using drawings to help intuition.

## PROOFS WITHOUT WORDS

This conception of mathematics was dominant at the age of what was called “modern mathematics” or “new math” (see [1]). Nevertheless, number of results has visual proofs (see [2]). The simplest of them is probably the calculation of the sum of the first natural numbers as:  $1 + 2 + 3 + 4 + 5$ . Of course, in this case, we find 15 easily but it will be more difficult to compute:  $1 + 2 + 3 + 4 + \dots + 100$ . The general case:  $1 + 2 + 3 + 4 + \dots + n$  is even more complex. The idea to compute it easily is to model this sum as the area of a staircase (see Figure 2).

Figure 2. Hervé Lehning,  $1 + 2 + 3 + 4 + 5 + 6$   
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By copying the staircase upside-down, we get a rectangle (see Figure 3).

Thus, twice the sum:  $1 + 2 + 3 + 4 + 5 + 6$  equals the area of the rectangle with side-lengths 6 and 7, which is 42 thus:  $1 + 2 + 3 + 4 + 5 + 6 = 21$ . For the same reason:  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ .

Today, this kind of proofs without words is generally accepted when they concern natural numbers (that is to say: 1, 2, 3, etc.). The same technique allows us to prove that the sum of the first  $n$  odd numbers equals the square of  $n$  (see Figure 4).

The identity:  $(a + b)^2 = a^2 + 2ab + b^2$  has a proof that, *a priori*, looks of the same kind. If the sides of the blue and orange squares are  $a$  and  $b$ , their areas equal  $a^2$  and  $b^2$  while those of the green rectangles equal the product  $ab$  (see Figure 5).

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