Chapter 19 Fuzzy Graphs and Fuzzy Hypergraphs

Michael G. Voskoglou

Graduate T. E. I. of Western Greece, Greece

Tarasankar Pramanik

b https://orcid.org/0000-0001-7582-2525 *Khanpur Gangche High School, India*

ABSTRACT

Relationship is the core building block of a network, and today's world advances through the complex networks. Graph theory deals with such problems more efficiently. But whenever vagueness or imprecision arises in such relationships, fuzzy graph theory helps. However, fuzzy hypergraphs are more advanced generalization of fuzzy graphs. Whenever there is a need to define multiary relationship rather than binary relationship, one can use fuzzy hypergraphs. In this chapter, interval-valued fuzzy hypergraph is discussed which is a generalization of fuzzy hypergraph. Several approaches to find shortest path between two given nodes in an interval-valued fuzzy graphs is described here. Many researchers have focused on fuzzy shortest path problem in a network due to its importance to many applications such as communications, routing, transportation, etc.

INTRODUCTION

Graph is a representation of relationship between objects. It can give a good idea of scope of relationship for any complex networking model. Graph has many variations such as directed graph, undirected graph, simple graph, pseudo graph, multi graph, finite graph, infinite graph, etc. In a directed graph the relation defined on V is not symmetric but in undirected graph the relation defined on V is symmetric. In a graph, loops may occur, that is, a vertex may have a relation to itself. Also, there may have more than one edges between two vertices, called parallel edges. Simple graphs have no multiple edges and loops at all. If in a graph, there are finite number of vertices and finite number of edges, then it is called finite graph otherwise, it is infinite graph. Most commonly, unless stated otherwise, graph means undirected

DOI: 10.4018/978-1-5225-9380-5.ch019

simple finite graph. In general, any mathematical problem involving points and connections among them can be called a graph and its pictorial representation may lead to a solution. Thus, graph as mathematical model of some problem can solve a graph-theoretic problem and then presents the solution of original problem.

Fuzzy graph is rather extension of (crisp) graph by introducing the concepts of fuzzy sets and fuzzy relations instead of (crisp) sets. The notion of fuzzy sets and fuzzy relations were first introduced by Rosenfeld (1975). After Rosenfeld, in 1977, Kaufmann (1973) introduced the notion of fuzzy hypergraphs.

FUZZY GRAPH

A fuzzy graph $\xi = (V, \sigma, \mu)$ is a triplet consisting of a non-empty set V together with a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that

$$\mu\left(xy\right) \leq \min\left\{\sigma\left(x\right), \sigma\left(y\right)\right\}$$

for all $x, y \in V$.

Here the two fuzzy sets σ and μ are called **fuzzy vertex set** and **fuzzy edge set** of ξ respectively. Clearly, μ is a fuzzy relation on σ .

The fuzzy subgraph of a fuzzy graph is a fuzzy graph whose fuzzy set is a subset of the fuzzy set of the given fuzzy graph.

A fuzzy graph $\xi = (V', \tau, \nu)$ is said to be a **partial fuzzy subgraph** of ξ if $\tau \subseteq \sigma$ and $\nu \subseteq \mu$.

The fuzzy graph $\xi = (V', \tau, \nu)$ is called a **fuzzy subgraph** of ξ if $\tau(x) \le \sigma(x)$ for all $x \in V'$ and $\nu(x, y) \le \mu(x, y)$ for all $x, y \in V'$ where $V' \subset V$.

A partial fuzzy subgraph $\xi' = (V', \tau, \nu)$ of ξ is said to span ξ if $\sigma = \tau$. This partial fuzzy subgraph ξ is called a spanning fuzzy subgraph of ξ .

An **underlying crisp graph** of a fuzzy graph $\xi = (V, \sigma, \mu)$ is a crisp graph $\xi' = (V, \sigma', \mu')$ where

$$\sigma' = \left\{ u \in V(\xi) \middle| \sigma(u) \right\} 0 \right\}$$

and

$$\mu' = \left\{ \left(u, v \right) \middle| \, \mu\left(u, v \right) \right\rangle 0 \right\}.$$

A path P in a fuzzy graph $\xi = (V, \sigma, \mu)$ is a sequence of distinct vertices $v_1, v_2, \dots, v_n (n \ge 2)$ such that

$$\mu\left(v_{_{i}},v_{_{i+1}}\right) > 0, i = 1,2,\ldots,\left(n-1\right).$$

30 more pages are available in the full version of this document, which may be purchased using the "Add to Cart" button on the publisher's webpage: www.igi-global.com/chapter/fuzzy-graphs-and-fuzzy-hypergraphs/235548

Related Content

Applying Graph Theory to Detect Cases of Money Laundering and Terrorism Financing

Natalia G. Miloslavskaya, Andrey Nikiforov, Kirill Plaksiyand Alexander Tolstoy (2020). Handbook of Research on Advanced Applications of Graph Theory in Modern Society (pp. 297-319). www.irma-international.org/chapter/applying-graph-theory-to-detect-cases-of-money-laundering-and-terrorism-financing/235541

Machine Learning Techniques Application: Social Media, Agriculture, and Scheduling in Distributed Systems

Karthikeyan P., Karunakaran Velswamy, Pon Harshavardhanan, Rajagopal R., JeyaKrishnan V.and Velliangiri S. (2020). *Handbook of Research on Applications and Implementations of Machine Learning Techniques (pp. 380-401).*

www.irma-international.org/chapter/machine-learning-techniques-application/234134

Demystification of Deep Learning-Driven Medical Image Processing and Its Impact on Future Biomedical Applications

R. Udendhranand Balamurugan M. (2020). *Deep Neural Networks for Multimodal Imaging and Biomedical Applications (pp. 155-171).*

www.irma-international.org/chapter/demystification-of-deep-learning-driven-medical-image-processing-and-its-impact-onfuture-biomedical-applications/259492

Identification of Optimal Process Parameters in Electro-Discharge Machining Using ANN and PSO

Kaushik Kumarand J. Paulo Davim (2022). *Research Anthology on Artificial Neural Network Applications* (*pp. 824-842*).

www.irma-international.org/chapter/identification-of-optimal-process-parameters-in-electro-discharge-machining-usingann-and-pso/288988

Reliability Study of Polymers

Amit Sachdevaand Pramod K. Singh (2020). AI Techniques for Reliability Prediction for Electronic Components (pp. 45-54).

www.irma-international.org/chapter/reliability-study-of-polymers/240489