Chapter 1
Domination Theory in Graphs

E. Sampathkumar
University of Mysore, India

L. Pushpalatha
Yuvaraja's College, India

ABSTRACT

The study of domination in graphs originated around 1850 with the problems of placing minimum number of queens or other chess pieces on an n x n chess board so as to cover/dominate every square. The rules of chess specify that in one move a queen can advance any number of squares horizontally, vertically, or diagonally as long as there are no other chess pieces in its way. In 1850 enthusiasts who studied the problem came to the correct conclusion that all the squares in an 8 x 8 chessboard can be dominated by five queens and five is the minimum such number. With very few exceptions (Rooks, Bishops), these problems still remain unsolved today. Let $G = (V,E)$ be a graph. A set $S \subset V$ is a dominating set of G if every vertex in $V–S$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set.

INTRODUCTION

The study of Domination in Graphs originated around 1850 with the problems of placing minimum number of queens or other chess pieces on an $n \times n$ chess board so as to cover/dominate every square. The rules of chess specify that in one move, a queen can advance any number of squares horizontally, vertically or diagonally as long as there are no other chess pieces in its way. In 1850 enthusiasts who studied the problem came to the correct conclusion that all the squares in an *8 x 8* chessboard can be dominated by five queens and five is the minimum such number. With very few exceptions (Rooks, Bishops), these problems still remain unsolved today.

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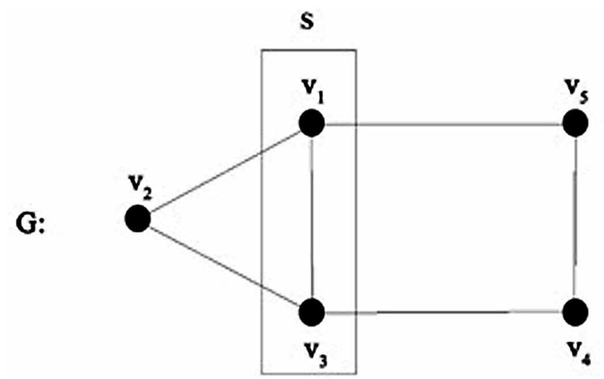
Figure 1.



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$S = \{v_1, v_3\}$ is a dominating set and $\gamma(G) = 2$.

APPLICATIONS

1. Berge (1973) in his book "Graphs and Hypergraphs" mentions the problem of keeping all points in a network under the surveillance of a set of radar stations. A number of strategic locations $v_1, v_2, \ldots$ called cells are kept under the surveillance of radar. Radar in cell v_2 can survey the locations v_1, v_2 or v_5. Similarly, v_3 can be surveyed by radar located at v_2 or v_3.

What is the minimum number of radar stations needed to survey all locations? It is the domination number of the network.

2. In a similar vein, Liu (1968) in his Book "Introduction to combinatorial Math," discusses the application of dominance to communication network, where a dominating set represents a set of cities which, acting as transmitting stations, can transmit messages to every city in the network.
3. Berge (1972) also discusses relationship between Kernels in graphs, i.e. dominating sets which are also independent and solutions to game theory.
4. The notion of dominance is used in coding theory. If one defines a graph whose vertices are n - dimensional vectors with coordinates chosen from (1, 2, …, p) and two vertices are adjacent if they differ in one coordinate, the sets of vectors which are (n, p) covering sets, or simple error correcting codes, or perfect covering sets are all dominating sets of the graph with certain additional properties. See for example, Kalbfleish, Stanton and Horton (1971), on covering sets and error correcting codes.