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Observations of Chaotic Behaviour in Nonlinear Inventory Models

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ABSTRACT

This article describes the use of simulation to investigate incipient chaotic behaviour in inventory models. Model structures investigated were either capacity limited or of variable delay time, implemented in discrete and continuous transform algebras. Results indicate the absence of chaos for a continuous time model but gave limited evidence for chaos in both unrestricted discrete models and those with a positive orders only limit. The responses where interaction with the capacity limit occurred did not confirm chaotic behaviour at odds with published results. Using the Liapunov exponent as a measure of chaotic behaviour, the results indicated, where the delay varies in proportion to order rate, a larger fixed delay reduced the Liapunov exponent as did increasing the dependence of delay on order rate. The effect of the model structures showed that the IOBPCS model, produced the largest Liapunov exponent. Reducing the discrete model update time reduced the Liapunov exponent.

KEYWORDS

APVIOBPCS, Chaos, Continuous Simulation, Discrete Simulation, Inventory, Liapunov Exponents, Order Rate Capacity Limits, Variable Delay Models

INTRODUCTION

In modern engineering, science and management great use is made of models to enable predictions to be made. Studies of the dynamics of physical and human systems are usually based on experiments and on the decision processes used to control them. Such studies of the variation with time depend on the initial conditions and on the parameters to be determined, usually by experiment but also by theoretical analysis. Some deterministic dynamic systems have been shown to be subject to chaotic behavior (Drazin, 1992). Fawcett and Waller (2011) have shown clearly why rigorous theoretical analysis of business processes is as important to progress as more practical discussions. They argue for dual approach but with rigorous evaluation of all research. This paper attempts to answer the question whether evidence for chaos in supply chains is real.

Chaos has been defined by Wilding (1998) as "…aperiodic, bounded dynamics in a deterministic system with sensitivity dependence on initial conditions, and has structure in phase space…" Wilding further outlines these terms as:

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- **Aperiodic:** The same state is never repeated twice;
- **Bounded:** On successive iterations, the state stays within a finite range and does not approach ± infinity;
- **Deterministic:** There is a definite rule with no random terms governing the dynamics;
- **Sensitivity to initial conditions:** Two points that are initially close will drift apart as time proceeds;
- **Structure in phase space:** Nonlinear systems are described by multi-dimensional vectors. The space in which these vectors lie is called a phase space or state space. The dimensions of this phase space are an integer (Abarbanel 1996). Chaotic systems display discernible patterns when viewed.

Chaos is defined as a deterministic behavior of a system governed by fixed rules involving no random elements. The sensitivity to initial conditions is such that a minor change in any variable may result in a totally different response. Thompson and Stewart (2002) state that chaos is unpredictable over long time scales because any two phase space trajectories starting close to a chaotic attractor will separate as they progress in time. The separation rate will depend on the largest Liapunov exponent (Kapitaniak, 1998) that is related to the system eigenvalues. The phenomenon is referred to as Deterministic Chaos.

Chaotic behavior has been found in cardiac systems (Garfinkel et al, 1992), stock market performance (Weis, 1992) and management of telephone exchanges (Erramilli and Forys, 1991).

When a model is created of the chosen system experimental measurements may be used. The ultimate accuracy of experiments is necessarily limited both by time and money and by the character of the parameters themselves. It is therefore imperative to determine what effect small changes in value of parameters have on the overall system performance. This not only has a bearing on whether the system as described behaves as needed but to see if the model is a true representation of the real system i.e. to see how sensitive the system is to parameter changes. A review of the history and progress in understanding chaos is given by Thompson and Stewart (2002).

In section 2 a review is presented of the published evidence that chaotic behavior exists in supply chains. The original purpose of this research was to examine whether the real limits in an inventory model were the cause of chaos observed in the behavior of inventory systems. This is followed by an examination of various models of inventory of the APVIOBPCS type (Lalwani et al., 2006) to see if they incur chaotic behavior. The results of these simulations will show which models are susceptible to chaos and some of the features that allow chaotic states to exist. Some of the observations may allow a possible path manipulation to reduce the occurrence of chaos. Simulation was chosen for this work since the conditions are controllable and repeatable whereas observing real supply chains have the problem of not being able to measure all the parameters involved and simulated models may not include all the real relevant elements. In this work we do not include the costs as these are usually specific to different products and companies and their inclusion would introduce a range of limits and conditions.

In this paper, we examine three specific areas of inventory operation where nonlinearity exists or may be introduced:

1. The approximations introduced in the implementation of a discrete model;
2. The effects of resource capacity limits in a system;
3. The introduction of variable time delays that may depend on other variables in the system, such as order rate.

Since the APVIOBPCS inventory models are examined in this paper, we exclude the possible case of nonlinear inventory or WIP gains, which are to be examined elsewhere.

In the rest of the paper a review is made of reported chaos in supply chains, followed by a discussion of the inventory models used in this work. The numerical experiments made are divided

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