

## Chapter 2

# Analysis of Two Phases Queue With Vacations and Breakdowns Under T-Policy

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### ABSTRACT

*The service system considered in this chapter is characterized by an unreliable server. Random breakdowns occur on the server and the repair may not be immediate. The authors assume the possibility that the server may take a vacation at the end of a given service completion. The server resumes operation according to T-policy to check if enough customers have arrived while he was away. The actual service of any arrival takes place in two consecutive phases. Both service phases are independent of each other. A Markov chain approach is used to obtain the steady state system size probabilities and different performance measures. The optimal value of the threshold level is obtained analytically.*

### INTRODUCTION

Waiting in line is an experience that everyone practices, almost, on a daily basis. The waiting takes different forms and settings. Nowadays, the competition of customer satisfaction and low cost becomes very intense to the point that any customer that waits too long in line is potentially a lost customer to another compactor that provides better service or better waiting environment.

Waiting lines are basic in structure to the external (customer-facing) and the internal business processes. Queueing structures generally include staffing, scheduling and inventory levels. For this reason, businesses often utilize queueing theory as a competitive advantage. Although queueing is undesired for anyone, it is the cornerstone of efficiency and organization for many companies. The idea is simple: At any given moment, there can be more people or cases needing service, help or attention than an organization can handle. Queues help workers and managers track, prioritize and ensure the delivery of services and transactions.

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The theory of waiting lines provides insight and identifies management options for improving customer service. A wide variety of queueing models have been developed and successfully exploited for very complex service situations. This chapter describes one such queueing model. For a comprehensive classification of various control policies applied in queueing systems, see the survey by Tadj and Choudhury (2005).

The service system considered in this chapter is characterized by an unreliable server. Random breakdowns occur on the server and the repair may not be immediate. It is assumed that at the end of a given service, the server may either take a vacation or start serving the next customer. When the queue is empty, the server again takes vacations and scans the queue periodically, every  $T$  units of time, to check if some customers have arrived while he was away. The third assumption is that the actual service of any arrival takes place in two consecutive phases. Both service phases are independent of each other.

## **BACKGROUND**

The unreliability of the server is one of the main features of the queueing system studied in this chapter. Queueing systems prone to failure are commonly encountered in the real world. The server breakdown was first analyzed by White and Christie (1958). Since then, queueing systems with unreliable servers have been extensively studied by many researchers; see Tadj et al. (2012) for a comprehensive survey on the subject.

The next feature of the system of interest in this chapter is the Bernoulli vacation schedule. The classical vacation scheme with Bernoulli service discipline was introduced and developed by Keilson and Servi (1986). Various aspects of Bernoulli vacation models have been discussed by a number of authors; see the survey of Ke et al. (2010).

The other important feature considered in our service system is the server  $T$ -policy. The M/G/1 queue with a  $T$ -policy was first studied by Heyman (1977). Many variants of the  $T$ -policy discipline model have been considered in the literature since then. There is no recent survey on the subject of  $T$ -policy discipline; however, some details of the latest contributions are listed here. Wang and Ke (2002) consider a single non-reliable server in the ordinary M/G/1 queueing system operating under the  $N$ -policy, the  $T$ -policy and the  $\text{Min}(N, T)$ -policy. They show that the optimal  $N$ -policy and the optimal  $\text{Min}(N, T)$ -policy are always superior to the optimal  $T$ -policy. Tadj (2003) studies an M/G/1 quorum queueing system under  $T$ -policy. The quorum or  $q$ -policy means that the server does not start service unless a specified number of customers are in the queue, and service is always rendered to groups of a fixed size. Ke (2005) studies an M/G/1 queueing system with an unreliable server, startup, and the following modified  $T$ -policy: After all the customers are served in the queue exhaustively, the server deactivates and takes at most  $J$  vacations of constant time length  $T$  repeatedly until at least one customer is found waiting in the queue upon returning from a vacation. If no customers arrive by the end of the  $J^{\text{th}}$  vacation, the server remains dormant in the system until at least one customer arrives. This model is generalized by Ke (2008) to the case of compound Poisson arrival process. More complex scenarios for the server are considered by Ke (2006). Kim and Moon (2006) study an M/G/1 queueing system where the server can take a vacation time  $T$  after the system becomes empty. At that time, the server is switched off with probability  $p$  and takes a vacation or remains on serving the arriving customers with probability  $(1-p)$ . Wang et al. (2009a) investigate the  $T$ -policy M/G/1 queue with server breakdowns and startup times. The server is turned on after a fixed length of time  $T$  repeatedly until at least one customer is present in the waiting line. The

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