Chaos in Oligopoly Models

Georges Sarafopoulos, Department of Economics, Democritus University of Thrace, Komotini, Greece

Kosmas Papadopoulos, Department of Economics, Democritus University of Thrace, Komotini, Greece

ABSTRACT

In this article, the authors investigate the dynamics of two oligopoly games. In the first game, they consider a nonlinear Cournot-type duopoly game with homogeneous goods and same rational expectations. The authors investigate the case, where managers have a variety of attitudes toward relative performance that are indexed by their type. In the second game they consider a Cournot-Bertrand duopoly game with linear demand, quadratic cost function and differentiated goods. In the two games they suppose a linear demand and a quadratic cost function. The games are modeled with a system of two difference equations. Existence and stability of equilibria of the systems are studied. The authors show that the models gives more complex, chaotic and unpredictable trajectories, as a consequence of change in the parameter k of speed of the player's adjustment (in the first game) and in the parameter d of the horizontal product differentiation (in the second game). The authors prove that the variation of the parameter k (resp. d) destabilizes the Nash equilibrium via a period doubling bifurcation (resp. through a Neimark-Sacker bifurcation). The chaotic features are justified numerically via computing Lyapunov numbers and sensitive dependence on initial conditions. In the second game they show that in the case of a quadratic cost there are stable trajectories and a higher or lower degree of product differentiation does not tend to destabilize the economy. They verify these results through numerical simulations. Finally, the authors control the chaotic behavior of the games introducing a new parameter. For some values of this parameter, the Nash equilibrium is stable for every value of the main parameter k or d.

KEYWORDS

Chaotic Behavior, Cournot – Bertrand Game, Cournot Duopoly Game, Discrete Dynamical System, Nash Equilibrium, Neimark-Sacker Bifurcation, Period Doubling Bifurcation, Stability

1. INTRODUCTION

An oligopoly is a market structure between monopoly and perfect competition, where there are only a few numbers of firms in the market producing homogeneous products. The dynamic of an oligopoly game is more complex, because firms must consider not only the behaviors of the consumers, but also the reactions of the competitors i.e. they form expectations concerning how their rivals will act. Cournot, in 1838, has introduced the first formal theory of oligopoly. He treated the case with naive expectations, so that, in every step, each player (firm) assumes the last values that were taken by the competitors, without estimation of their future reactions.

Traditional economic and industrial organization theories tend to view firms as entities, whose sole objective is to maximize their own profits. Recent researchers have criticized this view as an

oversimplification. Implicit in the profit maximization paradigm is the idea that managers care only about their own absolute performance, and that, consequently, they are unconcerned with how their performance compares to that of their rivals. We assume that managers have a variety of different attitudes toward relative performance, and that each manager's attitude is captured by his 'type' (Ting, Li, Junhai & Ma, 2015; Miller & Pazgal, 2002). In terms of their behavior, each manager will try to optimize some combination of his own profits and the rival firm's profits; his type determines the relative weights.

Expectations play an important role in modeling economic phenomena. A producer can choose his expectations rules out of many available techniques to adjust his production outputs. In this paper we study the dynamics of a duopoly model, where each firm behaves with homogeneous expectations strategies. (Agiza, 1999; Agiza et al., 2002; Agliari et al., 2005, 2006; Bischi & Kopel, 2001; Kopel, 1996; Puu, 1998; Sarafopoulos, 2015). Models with heterogeneous agents were also studied (Agiza & Elsadany, 2003, 2004; Agiza et al., 2002; Den Haan, 2013; Fanti & Gori, 2012; Tramontana, 2010; Zhang, 2007). Bounded rational players (firms) update their production strategies based on discrete time periods and by using a local estimate of the marginal utility. With such local adjustment mechanism, the players are not requested to have a complete knowledge of the demand function (Agiza & Elsadany, 2004; Naimzada & Sbragia, 2006; Zhang et al., 2007; Askar, 2014).

A large number of literatures about Cournot or Bertrand competition in oligopolistic market have been published, but there are only a considerably lower number of works devoted to Cournot Bertrand mixed competition, in which the market can be subdivided into two groups of firms. The first one optimally adjusts prices and the second optimally adjusts their outputs to ensure maximum profit. Cournot -Bertrand mixed models exist in realistic economy, and in some cases, Cournot- Bertrand competition may be optimal (Häckner, 2000; Zanchettin, 2006; Arya et al., 2008). For instance, in duopoly market, one firm competes in a dominant position, and it chooses output as decision variable, while the other one is in disadvantage, and it chooses price as decision variable in order to gain more market share. Recently Tremblay and Tremblay (2011) analyzed the role of product differentiation for the static properties of the Nash equilibrium of a Cournot–Bertrand duopoly. The present study extends Naimzada and Tramontana (2012).

In this study we investigate the dynamics of two oligopoly games. A Cournot- type duopoly game with homogeneous players and a Cournot-Bertrand game. The demand function is linear and the cost function is quadratic. We show that the models gives more complex, chaotic and unpredictable trajectories but in the first game the Nash equilibrium is destabilized through a period doubling bifurcation and in the second through a Neimark-Sacker bifurcation. In the second game we show that in the case of quadratic costs there are stable trajectories for each value of the differentiation parameter and a higher (lower) degree of product differentiation does not tend to destabilize the economy. We verify these results through numerical simulations. Introducing a new parameter we control the chaotic behavior of the two games.

The paper is organized as follows: In Section 2, the dynamics of the duopoly game with homogeneous expectations, linear demand and a quadratic cost function is analyzed. The existence and local stability of the equilibrium points are also analyzed. Numerical simulations are used to show complex dynamics via computing Lyapunov numbers (i.e. the natural logarithm of Lyapunov exponents), and sensitive dependence on initial conditions. We control the chaotic behavior introducing a new parameter. For some values of this parameter, the Nash equilibrium is stable for every value of our main parameter k (speed of adjustment) and we verify this result through numerical simulations.

In section 3, the dynamics of a Cournot – Bertrand duopoly model with linear demand, quadratic cost function and differentiated products is analyzed and we extends the work of Naimzada, Tramontana *(*Naimzada & Tramontana, 2012). Numerical simulations are used to show that the model gives more complex chaotic and unpredictable trajectories, as a consequence of change in the parameter d of horizontal product differentiation. We show the variation of this parameter destabilizes (stabilizes) the Nash equilibrium through Neimark-Sacker bifurcation. The chaotic features are justified numerically

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