Chapter XXXI
A 3D Environment for Exploring Algebraic Structure and Behavior

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ABSTRACT

Multimedia technology for personal computers has undergone a radical transformation over the past two decades with significant changes made in hardware-assisted audio and three-dimensional graphics processing. These changes suggest new possibilities for educational environments. The area of computer gaming represents one type of emerging technology since first- and third-person games have saturated the market over the past 10 years. We have taken an offshoot of computer games in the form of the multi-user, meta-gaming Second Life virtual environment, and explored its use for performing basic algebra operations such as the distributive property and computation of expressions. We leveraged the inherent multi-user collaborative-building capabilities within Second Life to explore how simple algebra manipulations can be accomplished. We also allowed students in a Spring 2007 class to build similar expressions as well as tool-based environments to facilitate the construction of virtual algebraic manipulatives. Our results suggest that while the current technology presents some key human interface challenges inherent to three-dimensional user interfaces, multi-user environments can be successfully used to construct algebraic expressions in ways not possible with prior technologies. Specifically, these environments provide real-time distance communication, the ability for multiple users to collaborate spatially toward creating and positioning algebraic components, sensory and cognitive immersion, and the possibility of personalizing representations in ways not easily accomplished with two-dimensional environments.

BACKGROUND AND MOTIVATION

We present a method and implementation for teaching the distributive law of algebra and basic algebraic computations within a multi-user environment called *Second Life* (Au et al., 2007; Cross, O'Driscoll, & Trondsen, 2007). The broader goal of our work is to avail ourselves of the technology of these environments and explore how they can be used to provide alternative modes of representation for basic structures in computer programming. Since algebraic structures are a fundamental part of the syntax of expressions within a computer program, our effort to date has centered on simple algebraic structures found in expressions that are located inside of program statements. With this goal in mind, several questions naturally arise:

1. Why are we using multi-user environments for algebra, or more generally, for computer programming?
2. How does this approach differ from other work performed in representing expressions and programs?
3. Why is *Second Life* relevant for this approach?
4. What is the role of assessment and evaluation?

We will take each of these questions in turn. To answer the first question, we must address it in three parts: (1) the importance of representation, (2) the role of technology, and (3) multi-user environments. If we take the following algebraic expression for the distributive law, $a * (b + c) = (a * b) + (a * c)$, we recognize this law in its common textual format. It might be difficult at first to imagine why we might want to explore such an expression in other formats. One of the key goals of mathematics education is to use multiple representations for the same concept or structure (NCTM, 2000). It is not necessarily that one representation is better than another, but that as a collection, alternate representations serve to provide a plurality of ways in which to think about a concept or structure. Since mathematical expressions and computer programs are types of languages, we can also look toward other areas such as media studies or new media for guidance (Manovich, 2002; Munster, 2006). The goal in media studies is to provide us with many ways in which to mediate the human experience. For example, one may learn about the history of a geographic region of interest by playing a board game, watching a video, reading a book, listening to a narrated audio script, or exploring a three-dimensional (3D) environment. These media forms are *complementary* rather than based on evolutionary replacement; it is both possible and advantageous to have a wide array of representations, with each type of representation offering a unique human perspective.

In response to the second part of the first question, we must address the role of technology. New representations are made possible and facilitated through technological advancement. The inventions of the printing press, cinema, and computer have contributed to new forms of representation not previously possible. One might argue that one should not utilize a technology for its own sake; however, the prudent path is to have some researchers engineering new environments for concepts to be learned (such as algebraic expressions), while other researchers attempt to establish boundary conditions for the new approach through assessment. As for addressing the third part of the first question, we note that multi-user environments are made possible through the new 3D gaming technology, and we should experiment with these environments to see how algebra can be represented and performed while fully exploiting the properties of these environments. Thus, if one has a 3D environment that offers a first-person perspective, a natural question would be how to represent variables, constants, and operators in three dimensions, and how to enable the distributive law with human interaction via an avatar.