Chapter 3
Portfolio Selection Models:
Single-Period Portfolio Selection

ABSTRACT

The main goal behind the concept of portfolio management is to combine various assets into portfolios and then to manage those portfolios so as to achieve the desired investment objectives. To be more specific, the investors' needs are mostly defined in terms of profit and risk, and the portfolio manager makes a sound decision aimed ether to maximize the return or minimize the risk. The Mean-Variance and Mean-VaR analysis has gained widespread acceptance among practitioners of asset allocation. Although they are the simplest models of investment, sometimes they are sufficiently rich to be directly useful in applied problems and decision theory. Here you will learn how to apply these analyses in practice using computer programs and spreadsheets.

A little over forty years ago, a University of Chicago graduate student in economics, while in search of a dissertation topic, ran into a stockbroker who suggested that he study the stock market. Harry Markowitz took that advice and developed a theory that became a foundation of financial economics and revolutionized investment practice. His work earned him a share of the 1990 Nobel Prize in Economics. (Paul D. Kaplan, 1998, Vice President and Chief Economist).

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MEAN-VARIANCE PORTFOLIO THEORY

In the finance textbooks, investment instrument that can be bought and sold is often called an asset. Asset allocation is a term used to describe the set of weights of broad classes of investment within a portfolio. For an individual investor, asset allocation can be represented by the investment in stocks, bonds, mutual funds and money market investments. Furthermore, the investor decisions are based on his utility function- level of welfare or satisfaction. The key reason for the utility function to be considered as a general approach to a decision making under risk is its sound theoretical basis.

Because future wealth is uncertain, investors attempt to maximize their expected value of utility. The relationship between wealth and the utility of consuming is described by a utility function, U (•). In general, each investor will have a different U (•), and we can write this formally as,

$$\max E[U(W) = \max E[U(W_1)] = E\{U[(1 + \sum_{i=1}^{N} X_i R_i)X_0 \quad \textit{subject to} \sum_i X_i = 1 \tag{1}$$

where, E represents the expected value of investors utility function U (•), and W is the wealth. U(•) is also called as von Neumann-Morgenstern utility function, based on the game theory work.

In Markowitz mean-variance portfolio theory (M-V), the rate of return of assets is random variable. The goal is then to choose the portfolio weighting factors optimally. Meaning, the investor's portfolio achieves an acceptable expected rate of return with minimal volatility. The variance of the rate of return is taken as a proxy for the volatility.

Let us now consider constructing a portfolio consisting of *n* assets. We have an initial budget $\boldsymbol{x_0}$that we wish to assign. The amount that we assign to asset *i* is $x_{oi} = w_i x_o$ for $i = 1, 2, \ldots . n$, where $\boldsymbol{w_i}$ is weighting factor for asset ***i***. What is also important to note, we allow the weights to take negative values. Here, when a negative value occurs, the asset is being shorted. Therefore, to preserve the budget constraint we require that the weights sum to be 1, $\sum_{i=1}^{n} w_1 = 1$. Thus, the sum of the investment is,

$$\sum_{i=1}^{n} w_1 x_0 = x_0 \qquad \sum_{i=1}^{n} w_1 = x_0 \tag{2}$$