

# Distributed Parameter Systems Control and Its Applications to Financial Engineering

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## INTRODUCTION

In several problems of financial engineering, such as options and commodities trading, forecasting of options' values, estimation of financial distress and credit risk assessment, validation of option pricing models, etc. one comes against Partial Differential Equations (PDEs). Moreover, in problems of control of financial systems where the aim is to stabilize financial processes which are described by PDE models, one has to harness again the complex PDE dynamics through the application of an external input. In the recent years differential flatness theory has emerged as an approach to the control and stabilization of systems described by PDE dynamics (Rudolph, 2003), (Rigatos, 2015). This research work focuses on differential flatness theory for the control and stabilization of single asset and multi-asset option price dynamics, described by PDE models. It is shown how the differential flatness approach achieves, stabilization of distributed parameter financial systems (that is systems modelled by PDEs) and how it enables convergence to specific financial performance indexes (Rigatos, 2014a; Rigatos, 2014b; Rigatos, 2014c; Rigatos, 2015a; Rigatos, 2015b; Rigatos, 2015c).

The Black-Scholes PDE is the principal financial model used in this study. It is demonstrated how with the use of semi-discretization and a finite differences scheme the single-asset (equivalently multi-asset) Black-Scholes PDE is transformed into a state-space model consisting of ordinary

nonlinear differential equations. For this set of differential equations it is proven that differential flatness properties hold (Rigatos, 2011; Rigatos, 2013; Rigatos, 2015). This permits to arrive at a solution for the associated control problem and to ascertain stabilization of the options' dynamics. By proving that it is feasible to control the single-asset (equivalently multi-asset) Black-Scholes PDE it is also concluded that through a selected trading policy, the price of options can be made to converge and stabilize at specific reference values.

The computational part of the considered feedback control method is as follows: For the local subsystems, into which the single-asset (equivalently multi-asset) Black-Scholes PDE is decomposed, it becomes possible to apply boundary-based feedback control. The controller design proceeds by showing that the state-space model of the single-asset (equivalently multi-asset) Black-Scholes PDE stands for a differentially flat system. Next, for each subsystem which is related to a nonlinear ODE, a virtual control input is computed, that can invert the subsystem's dynamics and can eliminate the subsystem's tracking error. From the last row of the state-space description, the control input (boundary condition) that is actually applied to the single-asset (equivalently multi-asset) Black-Scholes PDE system is found. This control input contains recursively all virtual control inputs which were computed for the individual ODE subsystems associated with the previous rows of the state-space equation. Thus, by tracing the rows of the state-space model

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backwards, at each iteration of the control algorithm, one can finally obtain the control input that should be applied to the single-asset (equivalently multi-asset) Black-Scholes PDE system so as to assure that all its state variables will converge to the desirable setpoints.

The structure of the chapter is as follows: in Section “*The problem of boundary control of the single-asset Black- Scholes PDE*”, an overview about the single-asset Black-Scholes PDE is given and the associated boundary control problem is formulated. In Section “*Option pricing modelling with the use of the single-asset Black-Scholes PDE*” the concept of option pricing for the single-asset Black-Scholes PDE is explained. In Section “*Transformation of the single-asset Black-Scholes PDE into nonlinear ODEs*” it is explained how the single-asset Black-Scholes PDE dynamics can be transformed to an equivalent state-space form. In Section “*Computation of boundary control for the single-asset Black-Scholes PDE*” a boundary feedback control law is computed for the single-asset Black-Scholes PDE. In Section “*Closed loop dynamics of the single-asset Black-Scholes PDE*” the dynamics of the closed control loop of the single-asset Black-Scholes PDE is analysed. In Section “*The problem of boundary control of the multi-asset Black-Scholes PDE*” the multi-asset Black-Scholes PDE is introduced and the associated boundary control problem is formulated. In Section “*Boundary control of the multi-asset Black-Scholes PDE*” it is explained how the multi-asset Black-Scholes PDE can be transformed into an equivalent state-space description. In Section “*Flatness-based control of the multi-asset Black-Scholes PDE*” it is analysed how a boundary feedback control input can be computed for the multi-asset Black-Scholes PDE. In Section “*Simulation tests*” the satisfactory performance of the control loop is confirmed through simulation experiments for both the single-asset and the multi-asset Black-Scholes PDE. Finally, in Section “*Conclusions*” concluding remarks are stated.

## **BACKGROUND**

One can note two modelling approaches for describing option price dynamics. The first one makes use of stochastic differential equations (SDEs). The underlying asset evolves according to a stochastic process that is driven by a random input. Using this representation, stochastic control methods for option price SDE diffusion models have been developed (Bensoussan, 2000), (Pascucci, 2011). Moreover, according to Kolmogorov’s theory, diffusion stochastic processes can be equivalently represented by partial differential equations (PDEs). These provide as solution the spatiotemporal distribution of the options’ value. The Black-Scholes PDE is such a relation (Platen & Heath, 2008; Sircar & Papanicolaou, 1998). Consequently, methods for PDE boundary control can be used for modifying option price dynamics. This topic will be elaborated in this article.

## **THE PROBLEM OF BOUNDARY CONTROL OF THE SINGLE-ASSET BLACK- SCHOLES PDE**

Control and stabilization of financial systems is a difficult problem since the associated models have spatiotemporal dynamics and are either described by partial differential equations or by stochastic differential equations (Platen & Heath, 2006; Pascucci, 2011). On the one side control approaches for financial systems have been developed with the use of stochastic differential equations (Oksendal & Sulem, 2006; Stojanovic, 2007; Yin et al., 2010). On the other side, control of financial dynamics through the use of the associated partial differential equations description remains an open problem for which efficient solutions have to be provided (Rudolph, 2003; Smyshlyaev & Krstic, 2010). To this end, in this research work a new control method is developed for the diffusion-type of the Black-Scholes PDE which describes the dynamics of options in financial markets. It is

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