Measures of Network Structure

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INTRODUCTION

Networked systems, natural or designed, have always been part of life. Their sophistication degree and complexity have increased through either natural evolution or technological progress. However, recent theoretical results have shown that a previously unexpected number of different classes of networks share similar network architectures and universal laws. Examples of such networks include metabolic pathways and ecosystems, the Internet and the World Wide Web, and organizational, social, and neural networks. Complex systems-related research questions investigated by researchers nowadays include: how consciousness arises out of the interactions of the neurons in the brain and between the brain and the environment (Amaral & Ottino, 2004; Barabási, 2005; Barabási & Oltvai, 2004; Neuman, 2003b) and how this understanding could be used for designing networked organizations or production networks whose behavior satisfies a given specification.

The aim of this article is to review the main measures of network structure and their applicability and to discuss future trends on this topic. The article is structured as follows. The Background section gives background information on networks. The Properties, Topologies, and Measures of Networks Structure section reviews the main properties and measures of network structure, mainly for graph-based representations of networks. It also includes several representative examples of networks. Next, the Discussion section briefly reviews the measures and their applicability, and the last section concludes the article.

BACKGROUND

Formally, this article focuses on large, complex networks characterized by the following properties:

- The networks have a large number of components, referred to as building blocks or agents, capable of interacting with each other and with the environment, and which may act according to rules that may change over time and that may not be well understood by an external observer.
- Homogeneous or heterogeneous, that is, identical or different agents, respectively.
- Hierarchical, modular, dynamically evolving structure, at component and connection level.
- Adaptability, that is, the network components respond to changes in the environment.
- Self-organization, where the network components make local decisions that have a coherent, organizing impact on the system as a whole. Therefore, the networks display organization without any external organizing principle being applied.
- Emergence, which is the process of complex pattern formation from simpler rules; emergent properties are neither properties had by any parts of the system taken in isolation nor a resultant of a mere summation of properties of parts of the system.
- Decision making based on local and/or global information and criteria.

Large-scale networks are highly dynamic, heterogeneous systems characterized by feedback-driven flow of information, openness, and emergence at the structural and behavioral levels (Amaral & Ottino, 2004; Prusinkiewicz & Lindenmayer, 2004). Strategic behavior and complex interactions take place at the node and subnetwork levels, as well as among the network components and the environment. The agents may constantly update their information about each other, about the system, and about the environment. They process this information to reason about other agents' information and thus to anticipate the actions and decisions of other agents and to finally use these predictions to make their own decisions.

Similar phenomena are encountered in many social situations and interactions in the field of security of communication, in artificial intelligence and distributed computing, as well as in the study of adaptive (both competitive and cooperative) behavior in biological populations. In such systems one often has a much simpler notion of a basic agent, and complex behavior is an emergent property of systems of such agents. Critical behavior is due to external attacks and to structural and/or behavioral failures. To understand such systems, one must understand how they interact, internally and with the environment. The challenge is to understand, model, and measure the structure and behavior of natural complex systems and to use this knowledge to design and manage complex systems that are autonomous; self-configurable, self-organized, and adaptive; robust; capable of self-recovery from critical conditions; scalable; deadlock-free; able to transfer knowledge and/or control to existing or new network members; flexible; and able to meet distributed demand in real time.

For example, metabolic pathways and ecosystems are biological and shaped by evolution. The Internet is a designed evolving network, and the propagation of HIV infection is a mixture of biology and sociological factors. Network theory has become a crucial theoretical support that has shown that all these networks, and many other natural and designed networks share similar network architectures (Amaral & Ottino, 2004; Barabási, 2003, 2005; Barabási & Oltvai, 2004; Boccaletti, Latora, Moreno, Chavez, & Hwang, 2006; Bornholdt & Schuster, 2003; da Costa, Travieso, & Ruggiero, 2005; Neuman, 2003a, 2003b).

PROPERTIES, TOPOLOGIES, AND MEASURES OF NETWORK STRUCTURE

Graph theory, statistical mechanics, and network theory are the representative approaches to modeling, analyzing, and measuring complex networks. The main structural properties and measures of networks are presented in this section (Ahuja, Magnanti, & Orlin, 1993; Barabási, 2003; Boccaletti et al., 2006; Bornholdt & Schuster, 2003). The terms graph and network are used interchangeably in the remainder of this article.

Graph Theory

Using graph theory, networks can be modeled as large directed or undirected graphs whose vertices and edges have different meanings depending on the context and application area (Ahuja et al., 1993; Newman, 2003b). A vertex represents the fundamental unit of a network, also called a site in physics, a node in computer science, or an actor in sociology. An edge represents the line connecting two vertices. The edge is also called a bond in physics, a link in computer science, or a tie in sociology.

An undirected (directed) graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ consists of the set of vertices or nodes, **V,** and the set of edges or links**, E**, such that $\mathbf{V} \neq \emptyset$ and **E** is a set of unordered (ordered) pairs of elements of **V**. Let us denote the number of vertices in **V** with N and the number of edges in **E** with E. Specific to graphs is the *adjacency (connectivity) matrix,* **A**, which is a NxN matrix whose entry a_{ij} is equal to 1 if *vertex i* and *vertex j* are adjacent and to 0 otherwise.

Network Topologies

Network topologies determine the rules of connecting the nodes and how nodes communicate with each other. There are two major types of network topologies: random networks and scale-free networks (Albert & Barabási, 2001; Barabási, 2003; Newman, 2003a, 2003b).

In random networks, the probability that two vertices are connected is random and uniform. Scale-free networks contain some important vertices called *hubs* that have a seemingly unlimited number of links and no vertex is typical of the others. Scale-free networks are remarkably resistant to accidental failures but extremely vulnerable to coordinated attacks. The scale-free model assumes that the network grows continuously by adding new vertices. New vertices would connect with higher probability to higher connected vertices, a phenomenon called *preferential attachment* (Albert & Barabási, 2001). Random networks are rather homogeneous, whereas the scale-free networks are extremely inhomogeneous.

An additional important concept is the *small-world* concept which is defined by the coexistence of two conditions: (1) despite their often large size, in most networks, there is a relatively short path between any two nodes (also referred to as the six degrees of separa-

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