

Logistic Models for Symbiosis, Predator–Prey, and Competition

Ricardo Lopez-Ruiz

Universidad de Zaragoza, Spain

Danièle Fournier-Prunaret

Institut National des Sciences Appliquées, France

If one isolated species (corporation) is supposed to evolve following the logistic mapping, then we are tempted to think that the dynamics of two species (corporations) can be expressed by a coupled system of two discrete logistic equations. As three basic relationships between two species are present in nature, namely symbiosis, predator-prey, and competition, three different models are obtained. Each model is a cubic two-dimensional discrete logistic-type equation with its own dynamical properties: stationary regime, periodicity, quasi-periodicity, and chaos. We also propose that these models could be useful for thinking in the different interactions happening in the economic world, as for instance for the competition and the collaboration between corporations. Furthermore, these models could be considered as the basic ingredients to construct more complex interactions in the ecological and economic networks.

INTRODUCTION

Malthus (1798) claimed that human population p grows faster than food supplies and that then poverty would be the inevitable result of overpopulation. He did not envisage the existence of a bound in the geometrical population growth given by the linear differential equation:

$$\frac{dp}{dt} = kp,$$

k being the growth rate. Verhulst (1845) argued that such limit is the consequence of the available resources: land and food. He established the simplest corrective hypothesis, namely that the growth co-efficient is pro-

portional to the distance of the population size from its saturation point. The result is an inhibitory term np^2 proportional to the square of the population size:

$$\frac{dp}{dt} = kp - np^2,$$

letting p tend asymptotically to the constant population k/n . Verhulst called this equation the *logistic function*. Until today, nobody knows the reason for this name although an explanation with the military meaning of the word “logistic” seems reasonable. We had to wait until the next century to see the importance of Verhulst’s work. It was the biologist Robert May (1976) who stated that the understanding of the discrete logistic model should be considered as a milestone in the field of non-linear phenomena. In concrete terms, the logistic equation is the basis of modern chaos theory, namely the paradigm for the period-doubling cascade (Feigenbaum, 1978) and also the paradigm for a particular type of intermittence (Pomeau & Manneville, 1980).

Let us establish the discrete logistic equation, that is, the discrete version of Verhulst model, to study the evolution of the population of a species (in ecology) or corporation (in economics). In the following, we restrict ourselves to the ecological interpretation, although all this work would be equivalent in the economic world where corporations interact as species do in an ecosystem. Let x_n represents the population of an isolated species after n generations and let us suppose that this variable is bounded in the range $0 < x_n < 1$. The discrete logistic evolution is given by the equation,

$$x_{n+1} = \mu x_n (1 - x_n), \quad (1)$$

where $0 < \mu < 4$ in order to assure $0 < x_n < 1$. The *activation or expanding phase* is controlled by the term

μx_n proportional to the current population x_n and to the constant growth rate μ . Resource limitations bring the system to an *inhibition or contracting phase* directly related with overpopulation. The term $(1-x_n)$ can denote how far the system is of overcrowding. Therefore, if we take the product of both terms as the simplest approach to the population dynamics, equation (1) results to be the equation modelling the rich dynamics of an isolated species with finite affordable resources. The dynamical behaviour that is found when the growth rate is modified is as follows (Collet & Eckmann, 1980; Mira, 1987):

1. $0 < \mu < 1$: The growth rate is not big enough to stabilize the population. It will drop and the specie will become extinct.
2. $0 < \mu < 3$: A drastic change is obtained when μ is greater than 1. The non vanishing equilibrium between the two competing forces, reproduction on one hand and resource limitation on the other, is now possible. The population reaches, independently of its initial conditions, a fixed value that is maintained in time.
3. $3 < \mu < 3.57$: A cascade of sudden changes provokes that the population oscillates in cycles of period 2^n , where n increases from 1, when μ is close to 3, to infinity when μ is approaching the critical value 3.57. This is named the period-doubling cascade.
4. $3.57 < \mu < 3.82$: When the parameter moves, the system alternates between periodical behaviours with high periods on parameter interval windows and *chaotic regimes* for parameter values not located in intervals. The population can be not predictable although the system is deterministic. The chaotic regimes are observed for a given value of μ on sub-intervals of $[0,1]$.
5. $3.82 < \mu < 3.85$: The orbit of period 3 appears for $\mu = 3.82$ after a regime where unpredictable bursts, named *intermittences*, have become rarer until their disappearance in the three-periodic time signal. The existence of the period 3 orbit means, such as the Sarkovskii theorem tells us, that all periods are possible for the population dynamics, although, in this case, they are not observable due to their instability. What is observed in this range is the period-doubling cascade $3 \cdot 2^n$.
6. $3.85 < \mu < 4$: Chaotic behaviour with periodic windows is observed in this interval.

7. $\mu = 4$: The chaotic regime is obtained on the whole interval $[0,1]$. This specific regime produces dynamics, which looks like random. The dynamics has lost near-all its determinism and the population evolves as a random number generator.

Therefore, there are essentially three remarkable dynamical behaviours in this system: the period doubling route to chaos around the value $\mu \approx 3.57$ (Feigenbaum, 1978), the time signal complexification by intermittence in the neighbourhood of $\mu \approx 3.82$ (Pomeau & Manneville, 1980) and the random-like dynamics for $\mu = 4$.

BACKGROUND

Let us suppose now, under a similar scheme of expansion/contraction, that two species (x_n, y_n) are now living together. Each one of them evolves following a logistic-type dynamics,

$$\begin{aligned} x_{n+1} &= \mu_x(y_n)x_n(1-x_n), \\ y_{n+1} &= \mu_y(x_n)y_n(1-y_n). \end{aligned} \quad (2)$$

The interaction between species causes the growth rate $\mu(z)$ to vary with time, then $\mu(z)$ depends on the population size of the others and on a positive constant λ , that measures the strength of the mutual interaction. The simplest choice for this growth rate can be a linear increasing μ_1 or decreasing μ_2 function expanding at the parameter interval where the logistic map shows some activity, that is, $\mu \in (1,4)$. Thus, we have,

$$\begin{aligned} \mu_1(z) &= \lambda(3z+1), \\ \mu_2(z) &= \lambda(-3z+4). \end{aligned}$$

Depending on the combination of both functions μ_1 and μ_2 we obtain three different models:

1. The *symbiosis* between species can be modelled by the symmetrical coupling meaning a mutual interacting benefit, then $\mu_x = \mu_y = \mu_1$ (López-Ruiz & Fournier-Prunaret, 2004).
2. The *predator-prey interaction* is based on the benefit/damage relationship established between the predator and prey, respectively, then $\mu_x = \mu_1$ and $\mu_y = \mu_2$ (López-Ruiz & Fournier-Prunaret, 2005).

8 more pages are available in the full version of this document, which may be purchased using the "Add to Cart" button on the publisher's webpage: www.igi-global.com/chapter/logistic-models-symbiosis-predator-prey/17697

Related Content

The 3-D Innovation Sphere: Exploring the Use of Second Life for Innovation Communication

Katrin Tobiesand Bettina Maisch (2012). *Virtual Worlds and Metaverse Platforms: New Communication and Identity Paradigms* (pp. 70-87).

www.irma-international.org/chapter/innovation-sphere-exploring-use-second/55400

Purchasing as an Integrated Competence

Erla M. Morales, Francisco J. Garcíaand Ángela Barrón (2008). *Encyclopedia of Networked and Virtual Organizations* (pp. 1309-1316).

www.irma-international.org/chapter/purchasing-integrated-competence/17758

Virtual Territorial Heritage in Education through mLearning Resources: Cities of Salamanca Spain and Santiago of Chile

Jorge Joo Nagata, José Rafael García-Bermejo Ginerand Fernando Martínez-Abad (2018). *Virtual and Augmented Reality: Concepts, Methodologies, Tools, and Applications* (pp. 345-377).

www.irma-international.org/chapter/virtual-territorial-heritage-in-education-through-mlearning-resources/199694

Information and Communication Technology (ICT) and Its Mixed Reality in the Learning Sphere: A South African Perspective

Ntokozo Mthembu (2018). *International Journal of Virtual and Augmented Reality* (pp. 26-37).

www.irma-international.org/article/information-and-communication-technology-ict-and-its-mixed-reality-in-the-learning-sphere/214987

Advanced Visual SLAM and Image Segmentation Techniques for Augmented Reality

Yirui Jiang, Trung Hieu Tranand Leon Williams (2022). *International Journal of Virtual and Augmented Reality* (pp. 1-28).

www.irma-international.org/article/advanced-visual-slam-and-image-segmentation-techniques-for-augmented-reality/307063