Fundamentals of Multirate Systems

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INTRODUCTION

Digital signal processing (DSP) is an area of science and engineering that has been rapidly developed over the past years. This rapid development is a result of significant advances in digital computers technology and integrated circuits fabrication (Mitra, 2005; Smith, 2002).

Classical digital signal processing structures belong to the class of single-rate systems since the sampling rates at all points of the system are the same.

The process of converting a signal from a given rate to a different rate is called sampling rate conversion. Systems that employ multiple sampling rates in the processing of digital signals are called multirate digital signal processing systems. Sample rate conversion is one of the main operations in a multirate system (Harris, 2004; Stearns, 2002).

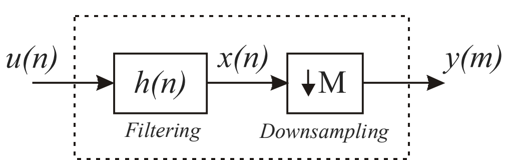
BACKGROUND

Decimation

The reduction of a sampling rate is called decimation, because the original sample set is reduced (decimated). Decimation consists of two stages: filtering and downsampling, as shown in Figure 1. The discrete input signal is *u(n)* and the signal after filtering is *x(n)*. Both signals have the same input sampling rate f_i.

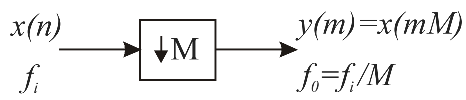
Downsampling reduces the input sampling rate f_i by an integer factor *M*, which is known as a downsampling factor. Thus, the output discrete signal *y(m)* has the sampling rate f_i/M. It is customary to use a box with a down-pointing arrow, followed by a downsampling factor as a symbol to represent downsampling, as shown in Figure 2.

Figure 1. Decimation



DECIMATION

Figure 2. Downsampling



The output signal *y(m)* is called a downsampled signal and is obtained by taking only every *M*-th sample of the input signal and discarding all others,

$$y(m) = x(mM). \qquad (1)$$

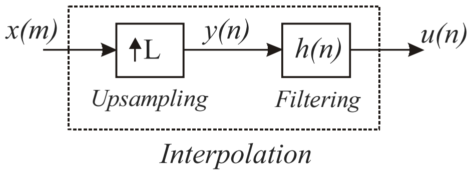
The operation of downsampling is not invertible because it requires setting some of the samples to zero. In other words, we can not recover *x(n)* from *y(m)* exactly, but can only compute an approximate value.

In spectral domain downsampling introduces the repeated replicas of the original spectrum at every $2\pi/M$. If the original signal is not bandlimited to π/M, the replicas will overlap. This overlapping effect is called aliasing. In order to avoid aliasing, it is necessary to limit the spectrum of the signal before downsampling to below π/M. This is why a lowpass digital filter (from Figure 1) precedes the downsampler. This filter is called a decimation or antialiasing filter.

Three useful identities summarize the important properties associated with downsampling (Jovanovic Dolecek, 2002). The First identity states that the sum of the scaled, individually downsampled signals is the same as the downsampled sum of these signals. This property follows directly from the principle of the superposition (linearity of operation). The Second identity establishes that a delay of *M* samples before the downsampler is equivalent to a delay of one sample after the downsampler, where *M* is the downsampling factor. The Third identity states that the filtering by the expanded filter followed by downsampling, is equivalent to having downsampling first, followed by the filtering with the original filter, where the expanded filter is obtained by replacing each delay of the original filter with *M* delays. In the time domain this is equivalent to inserting *M*-1 zeros between the consecutive samples of the impulse response.

The polyphase decimation, which utilizes polyphase components of a decimation filter, is a preferred structure for decimation, because it enables filtering to be performed at a lower sampling rate (Diniz, da Silva & Netto, 2002).

Figure 3. Interpolation



Interpolation

The procedure of increasing the sampling rate is called interpolation, and it consists of two stages: upsampling and filtering (shown in Figure 3).

The upsampler increases the sampling rate by an integer factor L, by inserting L-1 equally spaced zeros between each pair of samples of the input signal $x(n)$ as shown by

$$y(n) = \begin{cases} x(n/L) & for \;\; n = mL \\ 0 & otherwise \end{cases} \tag{2}$$

where L is called interpolation factor.

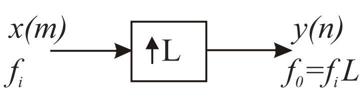
As Figure 4 illustrates, the symbol for this operation is a box with an upward-pointing arrow, followed by the interpolation factor. We can notice that the input sampling rate f_i is increased L times.

The process of upsampling does not change the content of the input signal, and it only introduces the scaling of the time axis by a factor L. Consequently, the operation of upsampling (unlike downsampling) is invertible, or in other words, it is possible to recover the input signal $x(m)$ from samples of $y(n)$ exactly.

The process of upsampling introduces the replicas of the main spectra at every $2\pi/L$. This is called imaging, since there are L-1 replicas (images) in 2π. In order to remove the unwanted image spectra, a lowpass filter must be placed immediately after upsampling (Figure 3). This filter is called an anti-imaging filter. In the time domain, the effect is that the zero-valued samples introduced by upsampler are filled with "interpolated" values. Because of this property, the filter is also called an interpolation filter.

We have already seen three useful identities of the downsampled signals, and now we will state the identities associated with upsampling. The Fourth identity asserts that the output signal obtained by upsampling followed by scaling of the input signal will give the same result as if the signal is first scaled and then upsampled. The Fifth identity states that a delay of one sample before upsampling is equivalent to the delay of L samples after upsampling. The Sixth identity, which is a more general version of the Fifth identity, states that filtering followed by upsampling is equivalent to having upsampling first followed by expanded filtering (Jovanovic Dolecek, 2002; Mitra, 2005; Diniz, da Silva & Netto, 2002).

Figure 4. Upsampling



Cascade of Sampling Converters

An interchange of cascaded sampling converters can often lead to a computationally more efficient realization (Fliedge, 2000; Vaidyanathan, 1993). If upsampling precedes downsampling, where both operations have the same factor, the signal is not changed. However, if downsampling is performed before upsampling, and both operations have the same factor, the resulting signal will be different from the input signal. Rational sampling conversion, that is, changing the sampling rate by a ratio of two integers, L/M can be efficiently performed as a cascade of upsampling and downsampling, where the interpolation and decimation filters are combined into one filter.

FUTURE TRENDS

Multirate systems have applications in digital radio, speech processing, telecommunications, wavelet transform, digital filtering, A/D converters, spectrum estimation, and so forth.

There are many applications where the signal of a given sampling rate needs to be converted into an equivalent signal with a different sampling rate. For example, in digital radio, three different sampling rates are used: 32 kHz in broadcasting, 44.1 kHz in digital compact disc (CD), and 48 kHz in digital audiotape (DAT), (Fliedge, 2000; Mitra, 2005). Conversion of the sampling rate of audio signals between these three different rates is often necessary. For example, if we wish to play CD music which has a rate of 44.1 kHz in a studio which operates at a 48 kHz rate, then the CD data rate must be increased to 48 kHz using a multirate technique.

In speech processing, multirate techniques are used to reduce the storage space or the transmission rate of speech data (Damiani, Dipanda, Yetongnon, Legrand, Schelkens & Chbeir, 2007; Meana, 2007). In the past years, multirate speech and audio signal processing has been a research topic that has produced several efficient algorithms for echo and noise cancellation, active noise cancellation, speech enhancement, and so forth. (Diniz, da Silva & Netto, 2002; Jovanovic Dolecek, 2002; Meana, 2007).

An example of an application of multirate signal processing in telecommunications is the translation between two multiplexing formats, time division multiplexing (TDM)

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