Radial Efficiency Measures in Data Envelopment Analysis

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INTRODUCTION

Data envelopment analysis (DEA) is a non-parametric mathematical programming methodology used to not only evaluate the relative efficiencies of comparable decision making units (DMUs) with multiple inputs and outputs, but also provide efficiency targets for the DMUs. For a visualization of this method, see Figure 1. Since the incept of Charnes, Cooper and Rhodes (1978), DEA has gained increasing popularity in theoretical and applied work. Cooper, Seiford and Tone (2006) provided an in-depth coverage of various DEA models. Cook and Seiford (2009) provided a sketch of the major research thrusts in DEA.

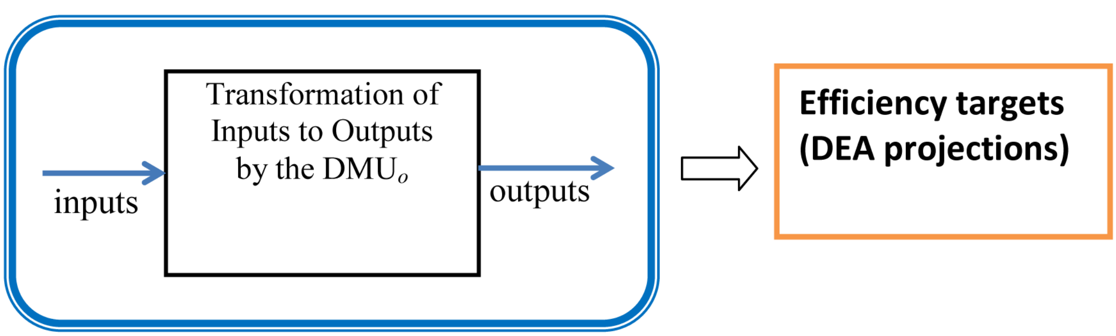
Examples of the DMUs used in real-world DEA applications include manufacturing firms, financial firms, and departments of organizations such as bank branches, hospitals, universities, schools and business firms, as well as regions and countries. Depending on whether inputs or outputs are discretionary or controllable input-oriented or output-oriented radial models can be employed. Both orientations are described in terms of radial efficiency measures as follows:

- Input orientation refers to the emphasis on the evaluated DMU's inputs that can be proportionately expanded without altering the output quantities.
- Output orientation refers to the emphasis on the evaluated DMU's outputs that can be proportionately reduced without altering the input quantities.

The term "radial efficiency" means that a proportional input reduction or a proportional output augmentation is the main concern in gauging and assessing the efficiency of the DMU. Hence, we can classify DEA models and measures[1] into four categories: (1) input-oriented and radial, (2) output-oriented and radial, (3) input-oriented and non-radial, (4) output-oriented and non-radial, and (5) other developments.

In addition to this classification, two scale assumptions are generally utilized: constant returns to scale (CRS) and variable returns to scale (VRS). The CRS assumption implies that a proportional increase in inputs results in a proportional increase in outputs. The VRS assumption implies that the DMUs to be analyzed exhibit increasing,

Figure 1. DEA efficiency targets for DMU$_o$



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decreasing or constant returns to scale. Although both CRS and VRS assumptions can be implemented in DEA, this chapter considers efficiency measures under the CRS[2] assumption, which is adopted by the original work of Charnes, Cooper and Rhodes (1978).

In this chapter, we focus on CRS input-oriented radial efficiency models (measures), in which the observed inputs are minimized given the level of outputs, and hence we deal with the category (1) mentioned above. We refer to the conventional input-oriented radial model (measure) as input-oriented CCR (Charnes-Cooper-Rhodes) or simply CCR model (measure).

The CCR measure evaluates the radial efficiency by searching a point on the weakly efficient frontier rather than the strongly efficient frontier[3], and hence the CCR projection point can be on a weakly efficient portion of the frontier. It follows that the CCR measure (the optimal objective function value of the CCR model) does not provide an efficiency score consistent to the strongly efficient frontier due to the use of a non-Archimedean infinitesimal number (not a real number in the standard mathematics), which is smaller than any positive real number (Cooper, Seiford, & Tone 2007, p.88). To find a strongly efficient projection point based on the CCR model, a two-step procedure is often employed. In this procedure, however, an efficiency score is not obtained. In order to make up for this deficiency, we resort to a recently developed measure by Fukuyama and Sekitani (2012a), which has several desirable features as an efficiency measure.

The axiomatic DEA literature starts with Färe and Lovell (1978). These authors proposed a set of desired properties that an input-oriented efficiency measure should satisfy. The Färe-Lovell properties[4] are indication, strong monotonicity and homogeneity: Indication states that the efficiency measure is equal to unity if and only if the input vector is Koopmans efficient; strong monotonicity shows that an increase in any input lowers the value of the measure, while holding the other inputs and all outputs fixed; and homogeneity states that if all inputs are multiplied by a positive constant, then the new value of the measure equals the original value multiplied by the reciprocal of the constant. Färe and Lovell (1978) demonstrated that the radial CCR measure (defined by the optimal radial factor) satisfies neither indication nor monotonicity. Russell (1985, 1987) pointed out that the so-called Russell measure[5] proposed by Färe and Lovell (1978) satisfies neither monotonicity nor homogeneity. Finally, Bol (1986) concluded that, for all general class of technologies satisfying minimum regularity conditions, there exists no efficiency measure that satisfies the Färe-Lovell properties. Taking these theoretical results into account, Dmitruk and Koshevoy (1991) provided a complete mathematical characterization for an efficiency measure to satisfy the properties. Russell and Shworm (2009) showed that even if the technologies are restricted to those generated by the most commonly used DEA (data envelopment analysis) methodologies, traditional DEA efficiency measures do not satisfy one or more of the desired properties. Recently, Fukuyama and Sekitani (2012a) proposed an efficiency measure defined relative to a DEA technology, and referred to it as eCCR (enhanced CCR). The eCCR measure satisfies indication, strong monotonicity and homogeneity. The purposes of this chapter are: (1) to analyze the CCR model and (2) to explain the eCCR measure.

DEA Models having Desirable Properties

Basics and the Input-Oriented CCR Model

Let $\mathbb{R}^L_+$ and $\mathbb{R}^L_{++}$ denote the L-dimensional non-negative and positive orthants, respectively. The conceptual production possibility set is defined by

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