Game Theory for Cost Allocation in Healthcare

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INTRODUCTION

Game theory is a branch of applied mathematics that studies strategic situations in which participants (players) act rationally in order to maximize their returns (payoffs). As such, game theory provides models of rational behavior (decision-making) for strategic interactions.

Many types of problems that involve decision strategies for cooperating or non-cooperating participants present a fruitful ground for application of mathematical game theory (Dowd, 2004; Cachon & Netessine, 2004).

In particular, cost allocation problems arise in many situations in which participants work together, such as healthcare providers who have to coordinate patient care in order to reduce the cost and improve quality of care. It was demonstrated that a natural framework for developing methodology for cost allocation problems could be based on game theoretical concepts (Tijs & Driessen, 1986; Roth, 1988; Young, 1994; Moulin, 2003). About a dozen of alternate concepts have been proposed to determine the 'fair' allocation but only a few of these concepts have received wide attention: the nucleolus and the Shapley value.

In this chapter these two concepts are compared. The focus is on demonstration of the practical application of the Shapley value for the cost allocation for cooperating providers. Two cases are illustrated:

1. The general application of the Shapley value methodology, and
2. An important particular case, in which each participant uses only a portion of the largest participant's asset (the so-called airport game).

BACKGROUND

By pooling resources and cooperating the participants usually reduce the total joint costs and realize savings. The question arises is how the reduced cost or the realized saving should be allocated fairly between them.

The simplest approach is dividing the cost reduction (savings) equally between all participants. However, this does not seem fair because the different contribution of each participant to the total gain. Another approach that looks fair is sharing the savings proportionally to the participants' own costs. However, the savings for some participants can be too low to keep them in voluntary cooperation with the bigger participants.

There could be different definitions of fair division. Some of them are:

- **Equitable Division:** Gives everyone the same satisfaction level, i.e. the proportion each player receives by their own valuation is the same for all of them. This is a difficult aim as players might not be truthful if asked their valuation.

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- **Proportional Division:** Guarantees that each player gets his share. For instance, if three people divide up an asset then each gets at least a third by their own valuation.
- **Envy-Free Division:** Everyone prefers his own share to the others. No one is jealous of anyone else. No one would trade his share with anyone else's.
- **An Efficient or Pareto Optimal Division:** Ensures that no other allocation would make someone better off without making someone else worse off. The term efficiency comes from the economics idea of the efficient market.

A concept of fairness is rather subjective. It depends on the participants' socio-economic views and other factors.

The fairness schemes described in the next section form a basis of the two most popular cost allocation approaches: the nucleolus (Tijs & Driessen, 1986; Saad, 2009) and the Shapley value (Roth, 1988; Yong, 1994).

MAIN FOCUS

The Nucleolus Concept

The nucleolus can be defined as an equilibrium that finds the 'center of gravity' of the so-called core. The core is defined as a set of inequalities that meet the requirement that no participant or a group of participants pays more than their stand-alone cost. The fairness criteria used by the nucleolus is minimizing the maximum "unhappiness" of a coalition. "Unhappiness" (or "excess") of a coalition is defined as the difference between what the members of the coalition could get by themselves and what they are actually getting if they accept the allocations suggested by the nucleolus.

More formally, an n-player game is defined by the set $N = \{1, 2, ...,n\}$ and a function $v(*)$, which for any subset gives a number $v(S)$ called the value of S. The characteristic value of the coalition S, denoted by $v(S)$, is the payoff that all players in the coalition S can jointly obtain. Let x_i be a payoff for player $i= 1, 2, \ldots,n$. The nucleolus solution is defined as $x = (x_1, x_2, \ldots, x_n)$ such that the excess ("unhappiness") $e_S(x) = v(S) - \sum x_i$ of any possible coalition S cannot be lowered without increasing any other greater excess. With this definition, the nucleolus is a solution that makes the largest "unhappiness" of the coalitions as small as possible.

There is no general closed-form formula for the nucleolus calculation, except for the recently developed analytic solution for a particular three-player case (Leng & Parlar, 2010). In general, the nucleolus has to be computed numerically in an iterative manner by solving a series of linear programming (LP) problems, or by solving a very large-scale LP problem. More specifically, the linear programming problem formulation is (Saad et al., 2009):

Z -> min

subject to:

$$Z + \sum_{i \in S} x_i \geq v(S)$$

$$\sum_{i \in N} x_i = v(N)$$

The advantage of the nucleolus is that it always exists, and that it is unique for all non-empty cores. Therefore, some researchers have used this concept to analyze business and management problems. As an early application of the nucleolus concept, Barton (1992) suggested the nucleolus solution as the mechanism to allocate joint costs among entities who share a common resource. At the same time, due to the complexity of the calculations for large coalitions, the nucleolus has not been extensively used to solve the various allocation-related problems.

Another problem with the nucleolus is that it does not exhibit the monotonicity property (Tijs & Driessen, 1986). Cost allocation concepts that

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