Data Envelopment Analysis for Operational Efficiency

Ramalingam Shanmugam
Texas State University, USA

INTRODUCTION

The *Data Envelopment Analysis* (DEA) is a popular operational research technique. This chapter explains and illustrates DEA. Building on the ideas of Farrell (1957) to describe the economic growths, the seminal work of DEA to measure the efficiency of the DMUs was pioneered by Charnes, Cooper & Rhodes (1978, 1979) using the linear programming (LP) concepts with the resources as inputs and products as outputs to make a *production frontier function*. Since then, there have been more than 3,000 publications on DEA or its applications in disciplines: *business, finance, police, trade, aviation, military, economics, sociology, farm, manufacturing, hospitals, oil refineries, banks, warehouses, restaurants, universities, service industries, global traders and environment protection agencies*. Feng et al. (1997) is an excellent book for LP.

The DEA is useful to institutions who want to improve their performance by decreasing the consumption of the inputs and/or increase the production of the outputs. The institution has to configure first how it is performing with respect to its competitors who are called *decision making units (DMUs)*. In DEA, the DMU are compared in terms of their *efficiency* which is defined to be the ratio of the produced total output to the consumed total input. When the total output is more per unit of the total input, the efficiency is high. An equivalent approach is that the efficiency is better when the total input is lesser per unit of an output. These two equivalent approaches are the basis for the *primal* and *dual* version of LP. Some reasons for efficiency assessment are: the board of directors might not approve an inefficient operation to continue, to make a future expansion or downsizing, the future dollars could be saved by minimizing the unused inputs which are identified in the process of rating, to make additional procurement of inputs or additional production of outputs, or to understand what good practices result in higher efficiency, to inform other competitors on how to flourish. The DEA's structure contains a linear *objective function*, *non-trivial* and *trivial constraints* in terms of the defined decision variables. The trivial constraints specify the non-negative nature of the decision variables. How much of a resource is used is a decision variable. The number of products is also a decision variable. Each decision variable personifies an output or input. The optimal value of every decision variable is identified in the best solution of a LP.

The number of transported patients to an emergency center of a hospital has to be either zero or positive but not negative and it is a trivial constraint. An example about a non-trivial constraint is the number of available beds for the inpatients occupancy in a given time at a hospital and it is recognized as the hospital's capacity. Realize that the inpatients might be different type depending on their medical needs. But, collectively their sum has to be less than or equal to the capacity of a hospital. It then constitutes a non-trivial resource constraint on the hospital's capacity. Likewise, for every resource, there is a constraint of less than or equal type as a part of non-trivial input constraints. With a similar logic, there will be a greater than or equal type non-trivial output constraint for every demand of the patients' service in a hospital. The objective function is a composition of all decision variables with their coefficients denoting the profit or cost amount per unit. The optimal values of the

DOI: 10.4018/978-1-4666-5202-6.ch053

decision variables are identified by solving the LP. The optimal value of the objective function indicates the maximum profit or the minimum cost with the intended outputs and usage of the inputs. In the case of profit, the objective function is maximized subject to satisfying the constraints. In the case of cost, the objective function is minimized. In a context of the medical applications, the objective function might be the transportation time from the patient's residence to a hospital in an ambulance emergency service and in which case, the objective function is minimized subject to satisfying the constraints. In the case of providing emergency service to the patients, there would be *n* decision variables (one for each area zone) in the objective function with their coefficients denoting the transportation time.

The first formulation of a LP is *primal* (PLP). An opposite version of the primal is *dual LP* (DLP). An optimal solution of the PLP and DLP is the same. The number of constraints in the primal is equal to the number of DMUs. The number of constraints in the dual equals the total number of inputs and outputs. More constraints would take a longer time to find optimal solution. Hence, either the primal or dual only, not necessarily both, is solved. Either the primal or dual of DEA is solved to quantify the efficiency scores of the DMUs. The DMUs are ranked using their *efficiency variable*, $\theta \in [0,1]$, which is a decision variable. In DEA, the objective function is just θ and it is minimized because θ denotes the fraction of the aggregated inputs of all the DMUs to be allocated to the chosen DMU in the assessment. The value one for θ means the chosen DMU is efficient. When $\theta < 1$, the chosen DMU is inefficient. The inefficiency is attributed to overusing the resources to obtain lesser outputs. A concern about DEA is that its optimal results could change when another DMU is included or a DMU is excluded and in which case a new analysis should be performed. The efficiency scores becomes a basis to halt the operation of an inefficient DMU until the improvements are done. A disadvantage of DEA is that the assessment of a DMU is relative only to other DMUs whose data are involved in the analysis. Hence, a great caution is necessary to put forward actions in practice based on DEA results. Still, DEA is a novel approach. When a particular DMU is inefficient, it can find who among other efficient DMUs to emulate based on the optimal DEA results. It is worthwhile to learn the evolutionary ideas of DEA in the next section.

LITERATURE REVIEW OF DEA

The chronological developments of DEA during 1970 - 1980s are documented in Ramanathan (2003), Sherman and Zhu (2006), Seiford and Thrall (1990), Thanassoulis (1995), and Ku-Mahamud et al (2011). Al-Najjar and Al-Jaybajy (2012) used DEA to rate the efficiency of the *oil refineries*. Ben-Arieh and Gullipalli (2012) used DEA to rate the *clinics*. Utilizing DEA, Yuksel (2012) rated the success of the *six sigma projects*. Ku-Mahamud et al (2011) has demonstrated how DEA helps to estimate efficiency of the projects. Using DEA, Shirouyehzad et al (2011) rated *safety* in the *construction projects*. Using DEA, Ozcan (2008) assessed how the clinics performed with respect to their *bench marks for the patients*. Farris et al (2006) has demonstrated the importance of DEA to evaluate the relative performance of *engineering design*s. Coelli et al (2005) illustrated the DEA to estimate the efficiency of the *production industries*. Using DEA, Paradi et al (2002) assessed the *workers' performance* at Bell Canada. Liu et al (2000) promotes the advantage of using DEA to compare suppliers in a *supply chain management*. Using DEA, Weber (1996) rated the *vendors' performance*. Bardhan et al (1988) rated production efficiency using DEA. Banker et al (1984) estimated the scale efficiencies of DEA. Banker (1980) used the *game theory* view of DEA to measure efficiency.

To be aware of the limitations of DEA, let us look at some critical thoughts. The DEA is a nonparametric technique. The parametric are

Related Content