Morphological Filtering Principles

Jose Crespo
Universidad Politécnica de Madrid, Spain

INTRODUCTION

In the last fifty years, approximately, advances in computers and the availability of images in digital form have made it possible to process and to analyze them in automatic (or semi-automatic) ways. Alongside with general signal processing, the discipline of image processing has acquired a great importance for practical applications as well as for theoretical investigations. Some general image processing references are (Castleman, 1979) (Rosenfeld & Kak, 1982) (Jain, 1989) (Pratt, 1991) (Haralick & Shapiro, 1992) (Russ, 2002) (Gonzalez & Woods, 2006).

Mathematical Morphology, which was founded by Serra and Matheron in the 1960s, has distinguished itself from other types of image processing in the sense that, among other aspects, has focused on the importance of shapes. The principles of Mathematical Morphology can be found in numerous references such as (Serra, 1982) (Serra, 1988) (Giardina & Dougherty, 1988) (Schmitt & Mattioli, 1993) (Maragos & Schafer, 1990) (Heijmans, 1994) (Soille, 2003) (Dougherty & Lotufo, 2003) (Ronse, 2005).

BACKGROUND

Morphological processing especially uses set-based approaches, and it is not frequency-based. This is in fact in sharp contrast with linear signal processing (Oppenheim, Schafer, & Buck, 1999), which deals mainly with the frequency content of an input signal. Let us mention also that Mathematical Morphology (as the name suggests) normally employs a mathematical formalism.

Morphological filtering is a type of image filtering that focuses on increasing transformations. Shapes can be satisfactorily processed by morphological filters. Starting with elementary transformations that are based on Minkowski set operations, other more complex transformations can be realized. The theory of morphological filtering is soundly based on mathematics.

This article provides an overview of morphological filtering. The main families of morphological filters are discussed, taking into consideration the possibility of computing hierarchical image simplifications. Both the binary (or set) and gray-level function frameworks are considered.

In the following of this section, some fundamental notions of morphological processing are discussed. The underlying algebraic structure and associated operations, which establish the distinguishing characteristics of morphological processing, are commented.

UNDERLYING ALGEBRAIC STRUCTURE AND BASIC OPERATIONS

In morphological processing, the underlying algebraic structure is a complete *lattice* (Serra, 1988). A complete lattice is a set of elements with a partial ordering relationship, which will be denoted as $\leq$, and with two operations defined called *supremum* (sup) and *infimum* (inf):

- The sup operation computes the smallest element that is larger than or equal to the operands. Thus, if a, b are two elements of a lattice, "a sup b" is the element of the lattice that is larger than both a and b, and there is no smaller element that is so.
- The inf operation computes the greatest element that is smaller than or equal to the operands.

Moreover, every subset of a lattice has an infimum element and a supremum element.

For sets and gray-level images, these operations are:

- Sets (or binary images)
  - o Order relationship: $\subseteq$ (set inclusion).
  - o "A sup B" is equal to "A $\cup$ B", where A and B are sets.
  - o "A inf B" is equal to "A $\cap$ B".

M

- Gray-level images (images with intensity values within a range of integers)
  - o Order relationship: For two functions *f,g*:

    $$f \leq g \Rightarrow f(x) \leq g(x),$$

    for all pixel *x*
    where the right-hand-side $\leq$ refers to the order relationship of integers.
  - o The sup of *f* and *g* is the function:

    $$(f \sup g)(x) = \max \{f(x), g(x)\}$$

    where "max" denotes the computation of the maximum of integers.
  - o The inf of *f* and *g* is the function:

    $$(f \inf g)(x) = \min \{f(x), g(x)\}$$

    where "min" symbolizes the computation of the minimum of integers.

TRANSFORMATION PROPERTIES

The concept of ordering is key in non-linear morphological processing, which focuses especially on those transformations that preserve ordering. An *increasing* transformation Ψ defined on a lattice satisfies that, for all *a,b*:

$$a \leq b \Rightarrow \Psi(a) \leq \Psi(b)$$

The following two properties concern the ordering between the input and the output. If *I* denotes an input image, an image operator Ψ is *extensive* if and only if, $\forall I$,

$$I \leq \Psi (I)$$

A related property is the anti-extensivity property. An operator Ψ is *anti-extensive* if and only if, $\forall I$,

$$I \geq \Psi (I)$$

The concept of *idempotence* is a fundamental notion in morphological image processing. An operator Ψ is idempotent if and only if, $\forall I$,

$$\Psi(I) = \Psi \Psi (I)$$

Within the non-linear morphological framework, the important *duality* principle states that, for each morphological operator, there exists a dual one with respect to the complementation operation.

Two operators Ψ and Ω are dual if

$$\Psi = C\Omega C$$

The complementation operation C, for sets, computes the complement of the input. In the case of gray-level images a related operation is the image inversion, which inverts an image reversing the intensity values with respect to the middle point of the intensity value range.

The following concept of *pyramid* applies to multi-scale transformations. A family of operators $\{\Psi_i\}$, where $i \in S = \{1,...,n\}$, forms a multi-level pyramid if

$$\forall j, k \in S, j \geq k, \exists l \text{ such that } \Psi_j = \Psi_l \Psi_k$$

In words, the set of transformations $\{\Psi_i\}$ constitutes a pyramid if any level *j* of the hierarchy can be reached by applying a member of $\{\Psi_i\}$ to a finer (smaller index) level *k*.

STRUCTURING ELEMENTS

A *structuring element* is a basic tool used by morphological operators to explore and to process the shapes and forms that are present in an input image. Normally, *flat* structuring elements, which are sets that define a shape, are employed. Two usual shapes (square and diamond) are displayed next (the "x" symbol denotes the center):
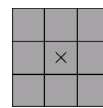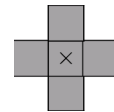
(a) Square 3x3   *(b) Diamond 3x3*

If *B* denotes a structuring element, its transposed is $\breve{B} = \{(-x,-y) \in B\}$ (i.e., *B* inverted with respect to the coordinate origin). If a structuring element *B* is centered and symmetric, then $\breve{B} = B$.

Related Content

Artificial Intelligence, Marketing, and the Fourth Industrial Revolution: Criteria, Concerns, Cases
Andreas Kaplan (2021). *Handbook of Research on Applied AI for International Business and Marketing Applications (pp. 1-13).*
www.irma-international.org/chapter/artificial-intelligence-marketing-and-the-fourth-industrial-revolution/261930

Inference Degradation of Active Information Fusion within Bayesian Network Models
Xiangyang Li (2008). *International Journal of Intelligent Information Technologies (pp. 1-17).*
www.irma-international.org/article/inference-degradation-active-information-fusion/2440

Software Effort Estimation: Harmonizing Algorithms and Domain Knowledge in an Integrated Data Mining Approach
Jeremiah D. Deng, Martin Purvisand Maryam Purvis (2013). *Organizational Efficiency through Intelligent Information Technologies (pp. 186-198).*
www.irma-international.org/chapter/software-effort-estimation/71968

Interval Type-2 Fuzzy Analytic Hierarchy Process for Sustainable Energy Sources Selection
Lazim Abdullahand Liana Najib (2017). *International Journal of Fuzzy System Applications (pp. 124-137).*
www.irma-international.org/article/interval-type-2-fuzzy-analytic-hierarchy-process-for-sustainable-energy-sources-selection/182229

Deep Learning-based Framework for Smart Sustainable Cities: A Case-study in Protection from Air Pollution
Nagarathna Ravi, Vimala Rani P, Rajesh Alias Harinarayan R, Mercy Shalinie S, Karthick Seshadriand Pariventhan P (2019). *International Journal of Intelligent Information Technologies (pp. 76-107).*
www.irma-international.org/article/deep-learning-based-framework-for-smart-sustainable-cities/237967