

# Many-Objective Evolutionary Optimisation

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## INTRODUCTION

Many-objective evolutionary optimisation is a recent research area that is concerned with the optimisation of problems consisting of a large number of performance criteria using evolutionary algorithms. Despite the tremendous development that multi-objective evolutionary algorithms (MOEAs) have undergone over the last decade, studies addressing problems consisting of a large number of objectives are still rare. The main reason is that these problems cause additional challenges with respect to low-dimensional ones. This chapter gives a detailed analysis of these challenges, provides a critical review of the traditional remedies and methods for the evolutionary optimisation of many-objective problems and presents the latest advances in this field.

## BACKGROUND

There has been considerable recent interest in the optimisation of problems consisting of more than three performance criteria, realm that was coined many-objective optimisation by Farina and Amato (Farina, & Amato, 2002). To date, the vast majority of the literature has focused on two and three-dimensional problems (Deb, 2001). However, in recent years, the incorporation of multiple indicators into the problem formulation has clearly emerged as a prerequisite for a sound approach in many engineering applications (Coello Coello, Van Veldhuizen, & Lamont, 2002). Despite the tremendous development that MOEAs have undergone over the last decade, and their ample success in disparate applications, studies addressing high-dimensional real-life problems are still rare (Coello Coello, & Aguirre, 2002). The main reason is that

many-objective problems cause additional challenges with respect to low-dimensional ones:

*If the dimensionality of the objective space increases, then in general, the dimensionality of the Pareto-optimal front also increases.*

*The number of points required to characterise the Pareto-optimal front increases exponentially with the number of objectives considered.*

It is clear that these two features represent a hindrance for most of the population-based methods, including MOEAs. In fact, in order to provide a good approximation of a high-dimensional optimal Pareto front, this class of algorithms must evolve populations of solutions of considerable size. This has a profound impact on their performance, since evaluating each individual solution may be a time-consuming task. Using smaller populations would not be a viable option, at least for Pareto-based algorithms, given the progressive loss of selective pressure they experience as the number of objectives increases, with a consequent deterioration of performances, as it is theoretically shown in (Farina, & Amato, 2004) and empirically evidenced in (Deb, 2001, pages 404-405). In contrast to Pareto-based methods, traditional multi-objective optimisation approaches, which work by reducing the multi-objective problem into a series of parameterised single-objective ones that are solved in succession, are not affected by the curse of dimensionality. However, such strategies cause each optimisation to be executed independent to each other, thereby losing the implicit parallelism of population-based multi-objective algorithms.

The remainder of this chapter provides a detailed review of the methods proposed to address the first two

issues affecting many-objective evolutionary optimisation and discusses the latest advances in the field.

### REMEDIAL MEASURES: STATE-OF-THE-ART

The possible remedies that have been proposed to address the issues arising in evolutionary many-objective optimisation can be broadly classified as follows:

- aggregation, goals and priorities
- conditions of optimality
- dimensionality reduction

In the next sub-sections we give an overview of each of these methods and review the approaches that have been so far proposed.

#### Aggregation, Goals and Priorities

This class of methods tries and overcome the difficulties described in the previous section through the decomposition of the original problem into a series of parameterised single-objective ones, that can then be solved by any classical or evolutionary algorithm.

Many aggregation-based methods have been presented so far and they are usually based on modifications of the weighted sum approach, such as the augmented Tchebycheff function, that are able to identify exposed solutions, and explore non-convex regions of the trade-off surface. However, the problem of selecting an effective strategy to vary weights or goals so that a representative approximation of the trade-off curve can be achieved is still unresolved.

The  $\epsilon$ -constraint approach (Chankong, & Haimes, 1983), which is based on minimisation of one (the most preferred or primary) objective function while considering the other objectives as constraints bound by some allowable levels, was also used in the context of evolutionary computing. The main limitation of this approach is its computational cost and the lack of an effective strategy to vary bound levels ( $\epsilon$ ). Recently, Laumanns *et al.* (Laumanns, Thiele, & Zitzler, 2006) proposed a variant of the original approach where they developed a variation scheme based on the concept of  $\epsilon$ -Pareto dominance (efficiency) (White, 1986) that adaptively generates constraint values, thus enabling the exhaustive exploration of the Pareto front, provided

the scheme is coupled with an exact single-objective optimiser. It must be pointed out however, that none of the methods described above has ever been thoroughly tested in the context of many-objective optimisation.

The Multiple Single Objective Pareto Sampling (MSOPS 1 & 2), an interesting hybridisation of the aggregation method with goal specification, was presented in (Hughes, 2003, Hughes, 2005). In the MSOPS, the selective pressure is not provided by Pareto ranking. Instead, a set of user defined target vectors is used in turn, in conjunction with an aggregation method, to evaluate the performance of each solution at every generation of a MOEA. The greater is the number of targets that a solution nears, the better its rank. The authors suggested two aggregation methods: the weighted min-max approach (implemented in MSOPS) and the Vector-Angle-Distance-Scaling (implemented in MSOPS 2). The results indicated with statistical significance that NSGA-II (Deb, Pratap, Agarwal, & Meyarivan, 2002), the Pareto-based MOEA used for comparative purposes, was outperformed on many objective problems. This was also recently confirmed by Wagner *et al.* in (Wagner, Beume, & Naujoks, 2007), where they benchmarked traditional MOEAs, aggregation-based methods and indicator-based methods on a up to 6-objective problems and suggested a more effective method to generate the target vectors.

#### Conditions of Optimality

Recently, great attention has been given to the role that conditions of optimality may play in the context of many-objective evolutionary optimisation when used to rank trial solutions during the selection stage of MOEA in alternative to, or conjunction with, Pareto efficiency. Farina *et al.* (Farina, & Amato, 2004) proposed the use of a fuzzy optimality condition, but did not provide a direct means to incorporate it into a MOEA. Köppen *et al.* (Köppen, Vincente-Garcia, & Nickolay, 2005) also suggested the fuzzification of the Pareto dominance relation, which was exploited within a generational elitist genetic algorithm on a synthetic MOP. The concept of *knee* (Deb, 2003), has also been exploited in the context of evolutionary many-objective optimisation. Simply stated, a knee is a portion of a Pareto surface where the marginal substitution rates are particularly high, i.e. a small improvement in one objective lead to a high deterioration of the others. A graphical representation is given in Figure 1. The idea

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