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Energy Minimizing Active Models in Artificial Vision

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INTRODUCTION

Deformable models are well known examples of artificially intelligent system (AIS). They have played an important role in the challenging problem of extracting useful information about regions and **areas of interest** (ROIs) imaged through different modalities. The challenge is also in extracting boundary elements belonging to the same ROI and integrate them into a coherent and consistent model of the structure. Traditional low-level image processing techniques that consider only local information can make incorrect assumptions during this integration process and generate unfeasible object boundaries. To solve this problem, deformable models were introduced (Ivins, 1994), (McInerney, 1996), (Wang, 2000). These AI models are currently important tools in many scientific disciplines and engineering applications (Duncan, 2000).

Deformable models offer a powerful approach to accommodate the significant variability of structures within a ROI over time and across different individuals. Therefore, they are able to segment, match and track images of structures by exploiting (bottom-up) constraints derived from the image data together with (top-down) *a priori* knowledge about the location, size, and shape of these structures.

The mathematical foundations of deformable models represent the confluence of geometry, physics and approximation theory. Geometry serves to represent object shape, physics imposes constraints on how the shape may vary over space and time, and optimal approximation theory provides the formal mechanisms for fitting the models to data. The physical interpretation views deformable models as elastic bodies which respond to applied force and constraints.

BACKGROUND

The deformable model that has attracted the most attention to date is the **active contour model** (ACM), well-known as **snakes**, presented by Kass *et al*. (Kass, 1987), (Cootes & Taylor, 1992). The mathematical basis present in snake models is similar to all deformable models, which are based on **energy minimizing** techniques.

Recently, there has been an increasing interest in level set or geodesic segmentation methods, introduced in (Osher & Sethian, 1988), (Malladi, 1995) and (Caselles, 1997). Level set approach involves solving the ACM minimization problem by the computation of minimal distances curve. This method allows topological changes within the ROIs and extension to 3D. Therefore, for some applications it is an improvement on classical ACM.

Other approaches to deformable model are those based on dynamic models or physically based techniques, for example superquadrics (Terzopoulos, 1991) and the finite element model (FEM) (Pentland, 1991). The FEM accurately describes changes in position, orientation and shape. The FEM can be used to solve fitting, interpolation or correspondence problems. In the FEM, interpolation functions are developed that allow continuous material properties, such as mass and stiffness, to be integrated across the ROIs. This last property makes them different from the previous models and therefore more suitable for some artificial vision applications.

The next sections contain a brief introduction to the mathematical foundations of deformable models.

ENERGY MINIMIZING DEFORMABLE MODELS

Geometrically, an active contour model is a parametric contour embedded in the image plane $(x, y) \in \mathbb{R}^2$. The dynamic contour is represented as a time-varying curve, $v(s,t) = (x(s,t), y(s,t))$, where x and y are the coordinate functions and $s \in [0, 1]$ is the parametric domain. The curve evolves until the ROI, subject to constraints from a given image $I(x, y)$, reaches an equilibrium. Thus, initially a curve is set around the ROI that, *via* minimization of an energy functional, moves normal to itself and stops at the boundary of the ROI. The energy functional is defined as:

$$E_{snake}(s,t) = \int_0^1 \left[E_{internal}(v(s,t)) + E_{ext\_potential}(v(s,t))\right] ds \tag{1}$$

The first term, $E_{internal}$, represents the internal energy of the spline curve due to mechanical properties of the contour, stretching and bending. It is a sum of two components, the elasticity and rigidity energy:

$$E_{internal}(s,t) = \left(\frac{\alpha(s,t)}{2} |v_s(s,t)|^2 + \frac{\beta(s,t)}{2} |v_{ss}(s,t)|^2 \right)$$

where α controls the tension of the contour, while β controls its rigidity. Thus, this functions determinate how the snake can stretch or bend at any point s of the spline curve. The second terms couples the snake to the image:

$$E_{ext\_potential}(s,t) = P(v(s,t))$$

where $P(v(s,t))$ denotes a scalar potential function defined on the image plane. It is responsible for attracting the contour towards the object in the image (external energy). Therefore, it can be expressed as a weighted combination of energy function.

To apply snakes to images, external potentials are designed whose local minima coincides with intensity *extrema*, edges and other image features of interest. For example, the contour will be attracted to intensity edges in an image by choosing a potential

$$P(v(s,t)) = -c\left|\nabla\left[G_\sigma * I(x,y)\right]\right|$$

where c controls the magnitude of the potential, ∇ is the gradient operator and $G_\sigma * I(x,y)$, denotes the image convolved with a Gaussian smoothing filter.

In accordance with the calculus of variations, the contour $v(s,t)$ that minimizes the energy of (1) must satisfy the Euler-Lagrange equation. Moreover, the Lagrange equation of motion for a snake with the internal and external energy given by equation (1) is:

$$\mu \frac{\partial^2 v}{\partial t^2} + \gamma \frac{\partial v}{\partial t} - \frac{\partial}{\partial s}\left(\frac{\alpha(s)}{2} |v_s(s,t)|^2 \right) + \frac{\partial^2}{\partial s^2}\left(\frac{\beta(s)}{2} |v_{ss}(s)|^2 \right) + \nabla P(v(s,t)) = 0$$

with a mass density μ and a damping density γ. This leads to dynamic deformable models that unify the description of shape and motion, making it possible to quantify not just static shape, but also shape evolution through time. The first two terms of this partial differential equation represent inertial and damping forces. The remaining terms represent the internal stretching, the bending forces and the external forces. Equilibrium is achieved when the internal and external forces balance and the contour comes to rest, i.e., inertial and damping forces are zero, which yields the equilibrium condition.

Traditional snake models are known to be limited in several aspects, such as their sensitivity to the initial contours. These are non-free parameters and do not handle changes in the topology of the shape. That is, when considering more than one object in the image, for instance for an initial prediction of $v(s,t)$ surrounding all of them, it is not possible to detect all the objects. Special topology-handling procedures must be added. Some techniques have been proposed to solve these drawbacks. These techniques are based on information

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