

Bayesian Neural Networks for Image Restoration

Radu Mutihac

University of Bucharest, Romania

INTRODUCTION

Numerical methods commonly employed to convert experimental data into interpretable images and spectra commonly rely on straightforward transforms, such as the Fourier transform (FT), or quite elaborated emerging classes of transforms, like wavelets (Meyer, 1993; Mallat, 2000), wedgelets (Donoho, 1996), ridgelets (Candes, 1998), and so forth. Yet experimental data are incomplete and noisy due to the limiting constraints of digital data recording and the finite acquisition time. The pitfall of most transforms is that imperfect data are directly transferred into the transform domain along with the signals of interest. The traditional approach to data processing in the transform domain is to ignore any imperfections in data, set to zero any unmeasured data points, and then proceed as if data were perfect.

Contrarily, the maximum entropy (ME) principle needs to proceed from frequency domain to space (time) domain. The ME techniques are used in data analysis mostly to reconstruct positive distributions, such as images and spectra, from blurred, noisy, and/or corrupted data. The ME methods may be developed on axiomatic foundations based on the probability calculus that has a special status as the only internally consistent language of inference (Skilling 1989; Daniell 1994). Within its framework, positive distributions ought to be assigned probabilities derived from their entropy.

Bayesian statistics provides a unifying and self-consistent framework for data modeling. Bayesian modeling deals naturally with uncertainty in data explained by marginalization in predictions of other variables. Data overfitting and poor generalization are alleviated by incorporating the principle of Occam's razor, which controls model complexity and set the preference for simple models (MacKay, 1992). Bayesian inference satisfies the likelihood principle (Berger, 1985) in the sense that inferences depend only on the probabilities assigned to data that were measured and not on the properties of some admissible data that had never been acquired.

Artificial neural networks (ANNs) can be conceptualized as highly flexible multivariate regression and multiclass classification non-linear models. However, over-flexible ANNs may discover non-existent correlations in data. Bayesian decision theory provides means to infer how flexible a model is warranted by data and suppresses the tendency to assess spurious structure in data. Any probabilistic treatment of images depends on the knowledge of the point spread function (PSF) of the imaging equipment, and the assumptions on noise, image statistics, and prior knowledge. Contrarily, the neural approach only requires relevant training examples where true scenes are known, irrespective of our inability or bias to express prior distributions. Trained ANNs are much faster image restoration means, especially in the case of strong implicit priors in the data, nonlinearity, and nonstationarity. The most remarkable work in Bayesian neural modeling was carried out by MacKay (1992, 2003) and Neal (1994, 1996), who theoretically set up the framework of Bayesian learning for adaptive models.

BACKGROUND

Bayesian approach to image restoration is based on the assumption that all of the relevant image information may be stated in probabilistic terms and prior probabilities are known. The ME principle is optimally setting prior probabilities for positive additive distributions. Yet Bayes' theorem and the ME principle share one common future: the updating of a state of knowledge. In some cases, running Bayes' theorem in one hypothesis space and applying the ME principle in another lead to similar calculations.

Neuromorphic and Bayesian modeling may apparently look like extremes of the data modeling spectrum. ANNs are non-linear parallel computational devices endowed with gradient descent algorithms trained by example to solve prediction and classification problems. In contrast, Bayesian statistics is based on coherent

inference and clear axioms. Yet both approaches aim to create models in agreement with data. Bayesian decision theory provides intrinsic means to model ranking. Bayesian inference for ANNs can be implemented numerically by deterministic methods involving Gaussian approximations (MacKay, 1992), or by Monte-Carlo methods (Neal, 1996). Two features distinguish the Bayesian approach to learning models from data. First, beliefs derived from background knowledge are used to select a prior probability distribution for model parameters. Secondly, predictions of future observations are performed by integrating the model's predictions with respect to the posterior parameter distribution obtained by updating this prior with new data. Both aspects are difficult in neural modeling: the prior over network parameters has no obvious relation to prior knowledge, and integration over the posterior is computationally demanding. The properties of priors can be elucidated by defining classes of prior distributions for net parameters that reach sensible limits as the net size goes to infinity (Neal, 1994). The problem of integrating over the posterior can be solved using Markov chain Monte Carlo (Neal 1996).

Bayesian Image Modeling

The fundamental concept of Bayesian analysis is that the plausibility of alternative hypotheses $\{H_i\}_{i \in \mathbb{N}}$ is represented by probabilities $\{P_i\}_{i \in \mathbb{N}}$, and inference is performed by evaluating these probabilities. Inference may operate on various propositions related in neural modeling to different paradigms. Bayes' theorem makes no reference to any sample or hypothesis space, neither it determines the numerical value of any probability directly from available information. As a prerequisite to apply Bayes' theorem, a principle to cast available information into numerical values is needed.

In statistical restoration of gray-level digital images, the basic assumption is that there exists a scene adequately represented by an orderly array of N pixels. The task is to infer reliable statistical descriptions of images, which are gray-scale digitized pictures and stored as an array of integers representing the intensity of gray level in each pixel. Then the shape of any positive, additive image can be directly identified with a probability distribution. The image is conceived as an outcome of a random vector $\mathbf{f} = \{f_1, f_2, \dots, f_N\}$, given in the form

of a positive, additive probability density function.

Likewise, the measured data $\mathbf{g} = \{g_1, g_2, \dots, g_M\}$ are expressed in the form of a probability distribution (Fig. 1). Further assumption refers to image data as a linear function of physical intensity, and that the errors (noise) \mathbf{b} is data independent, additive, and Gaussian with zero mean and known standard deviation σ_m , $m = 1, 2, \dots, M$ in each pixel. The concept of image entropy and the entropy alternative expressions used in image restoration are discussed by Gull and Skilling (1985). A brief review of different approaches based on ME principle, as well as a full Bayesian approach for solving inverse problems are due to Djafari (1995).

Image models are derived on the basis of intuitive ideas and observations of real images, and have to comply with certain criteria of invariance, that is, operations on images should not affect their likelihood. Each model comprises a hypothesis H with some free parameters $\mathbf{w} = (\alpha, \beta, \dots)$ that assign a probability density $P(\mathbf{f} | \mathbf{w}, H)$ over the entire image space and normalized to integrate to unity. Prior beliefs about the validity of H before data acquisition are embedded in $P(H)$. Extreme choices for $P(H)$ only may exceed the evidence $P(\mathbf{f} | H)$, thus the plausibility $P(H | \mathbf{f})$ of H is given essentially by the evidence $P(\mathbf{f} | H)$ of the image \mathbf{f} . Consequently, objective means for comparing various hypotheses exist.

Initially, the free parameters \mathbf{w} are either unknown or they are assigned very wide prior distributions. The task is to search for the best fit parameter set \mathbf{w}_{MP} , which has the largest likelihood given the image. Following Bayes' theorem:

$$P(\mathbf{w} | \mathbf{f}, H) = \frac{P(\mathbf{f} | \mathbf{w}, H) \cdot P(\mathbf{w} | H)}{P(\mathbf{f} | H)} \quad (1)$$

where $P(\mathbf{f} | \mathbf{w}, H)$ is the likelihood of the image \mathbf{f} given \mathbf{w} , $P(\mathbf{w} | H)$ is the *prior* distribution of \mathbf{w} , and $P(\mathbf{f} | H)$ is the evidence for H . A prior $P(\mathbf{w} | H)$ has to be assigned quite subjectively based on our beliefs about images. Since $P(\mathbf{w} | \mathbf{f}, H)$ is normalized to 1, then the denominator in (1) ought to satisfy $P(\mathbf{f} | H) = \int_{\mathbf{w}} P(\mathbf{f} | \mathbf{w}, H) \cdot P(\mathbf{w} | H) \cdot d\mathbf{w}$. The inte-

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